北京航空航天大學 BEIHANG UNIVERSITY

关键基础设施可靠性安全性研究中心 Center for Resilience and Safety of Critical Infrastructures

Belief Reliability for Uncertain Random Systems

Rui Kang

Center for Resilience and Safety of Critical Infrastructures
School of Reliability and Systems Engineering
Beihang University, Beijing, China

IEEE Reliability Society

QRS 2018

The 18th IEEE International Conference on
Software Quality, Reliability, and Security

July 16-20, 2018 • Lisbon, Portugal
http://paris.utdallas.edu/qrs18

A short introduction

School of Reliability and Systems Engineering, BUAA

Education
90 undergraduates every year
150 graduate students every year
40 Phd. candidates every year
120 faculty members

Research
More than 100 scientific research and hi-tech projects every year

Consultation
As a national think tank, provides policy advice to the government on reliability technology and engineering

Engineering
Provide a large number of technical services for industry

- National Key Laboratory for Reliability and Environmental Engineering
- Department of Systems Engineering of Engineering Technology
- Department of System Safety and Reliability Engineering
- Center for Product Environment Engineering
- Center for Components Quality Engineering
- Center for Software Dependability Engineering

Double Helix Structure of Reliability Science

Abstract Objects

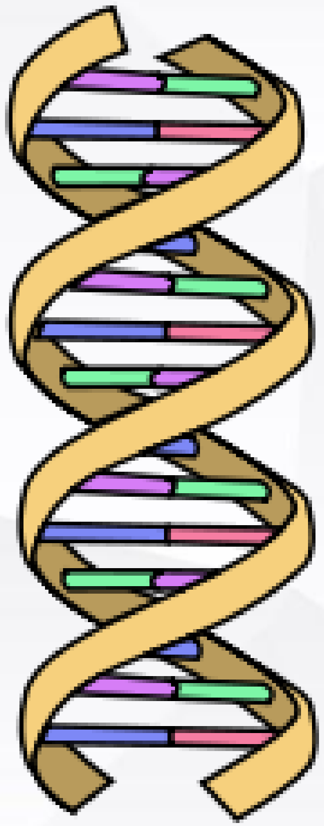
Cyber Physics Social System

Cyber Physics System

Network

Hardware+Software

Hardware & Software



Methodology

Failure/Fault Prophylaxis

Failure/Fault Diagnostics

Failure/Fault Prognostics

Failure/Fault Cebernetics

Failurology

Recognize Failure Rules & Identify Failure Behaviors

Outline

Research Background

Requirements Analysis

Theoretical Framework

Conclusion & Future

Outline

Research Background

Requirements Analysis

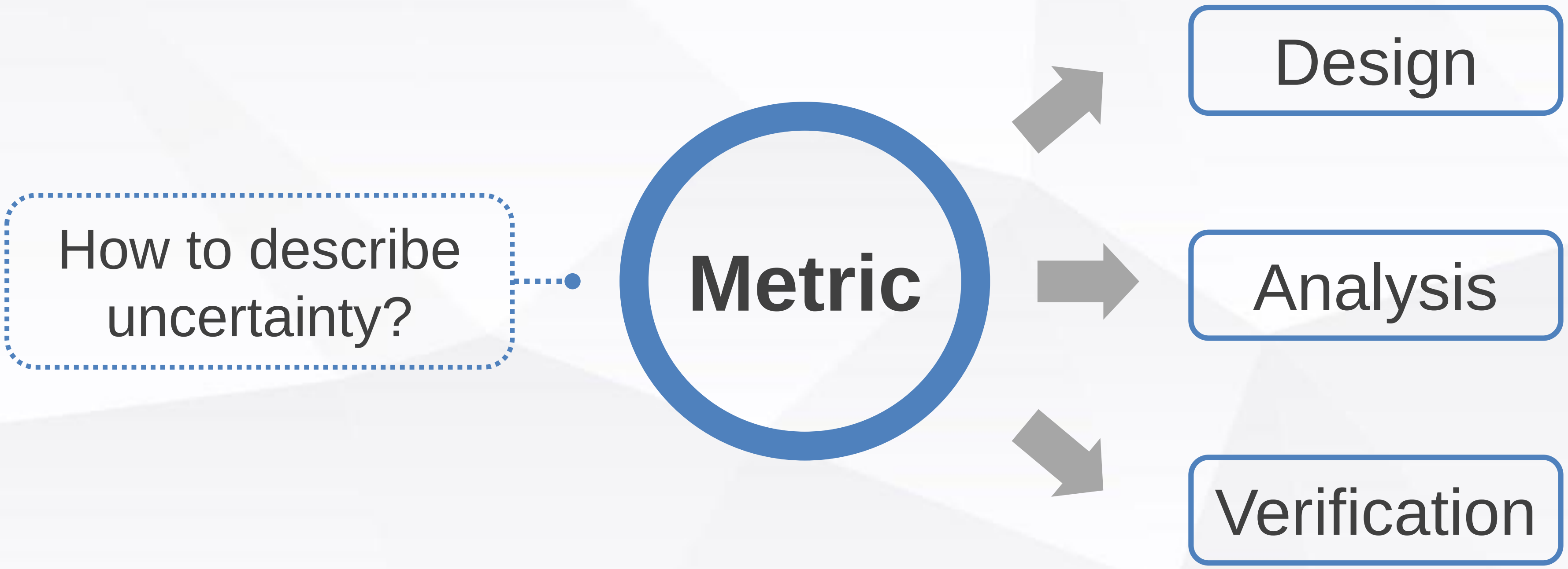
Theoretical Framework

Conclusion & Future

Reliability

Definition: Reliability refers to the ability of a component or a system to perform its required functions under stated operating conditions for a specified period of time.

Four basic problems: Reliability metric, analysis, design and verification



Uncertainty

Classification: Aleatory uncertainty & Epistemic uncertainty

Aleatory uncertainty

Inherent randomness of the physical world and can not be eliminated. This kind of uncertainty is also called random uncertainty.

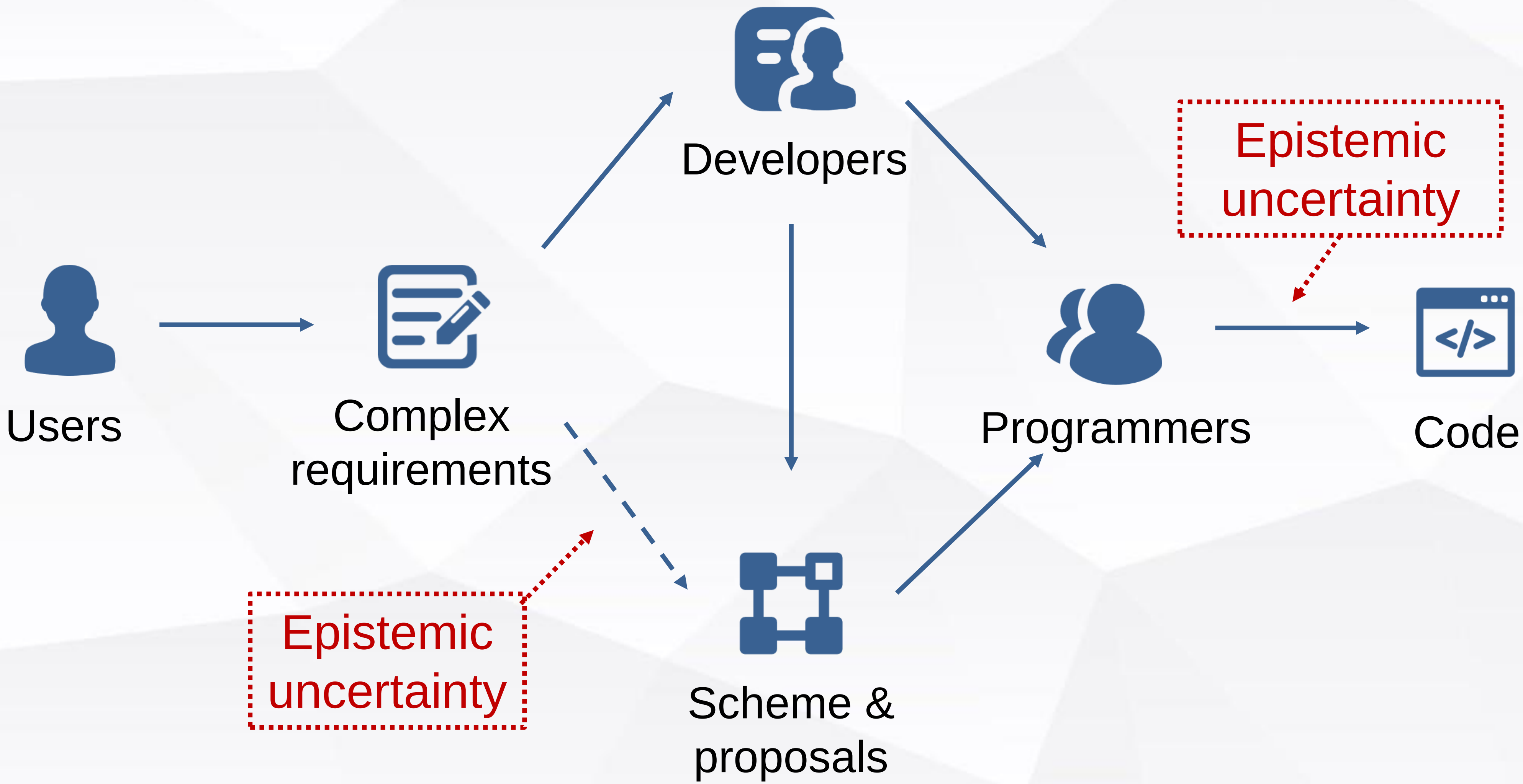
Epistemic uncertainty

Uncertainty due to lack of knowledge. It can be reduced through scientific and engineering practices.

[1] Kiureghian, Armen Der, and O. Ditlevsen. Aleatory or epistemic? Does it matter?. *Structural Safety* 31.2(2009): 105-112.

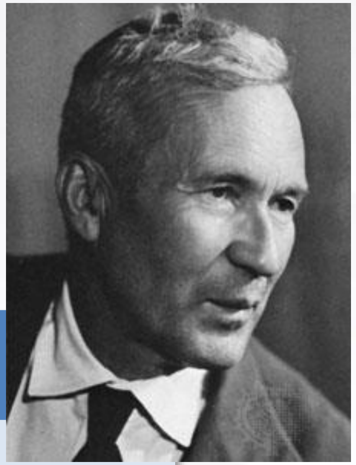
Source of epistemic uncertainty

Example - Software



Probability theory

Probability measure



Probability Theory（Kolmogorov,1933）

Axiom1. Normality Axiom: For the universal set Ω, $\Pr\{\Omega\} = 1$.

Axiom2. Nonnegativity Axiom: For any event A, $\Pr\{A\} \geq 0$.

Axiom3. Additivity Axiom: For every countable sequence of mutually disjoint events $\{A_i\}$, we have

$$\Pr\left\{\bigcup_{k=1}^{\infty} A_i\right\} = \sum_{k=1}^{\infty} \Pr\{A_i\}.$$

Product Probability Theorem: For any probability space $(\Omega_k, \mathcal{A}_k, \Pr_k), k = 1,2, \ldots$,

$$\Pr\left\{\prod_{k=1}^{\infty} A_k\right\} = \prod_{k=1}^{\infty} \Pr_k\{A_k\}.$$

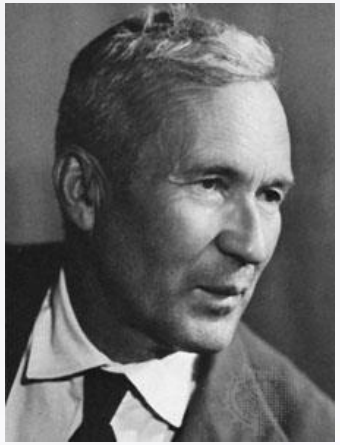
where A_k are arbitrarily chosen events from $\mathcal{A}_k, k = 1,2, \ldots$

Probability theory

The law of large numbers

J. Bernoulli

P. Chebyshev



A. Kolmogorov

Bernoulli's Law of Large Numbers（Bernoulli,1713）

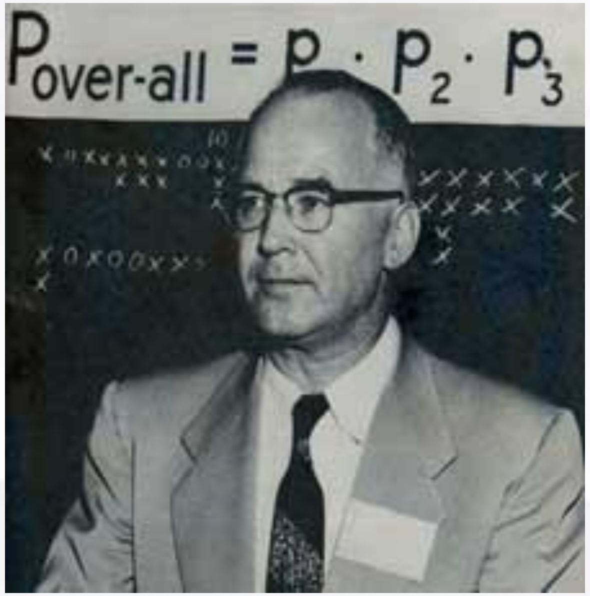
Let μ be the occurrence times of event A in n independent experiments. If the probability that event A occurs in each test is p, then for any positive number ε:

$$\lim_{n\to\infty} \Pr\left\{\left|\frac{\mu}{n} - p\right| < \varepsilon\right\} = 1.$$

Classical probabilistic reliability metric

At the very beginning…

- Probability theory is used to represent uncertainty
- In World War II, German rocket scientist Robert Lusser advocated the probability product rule



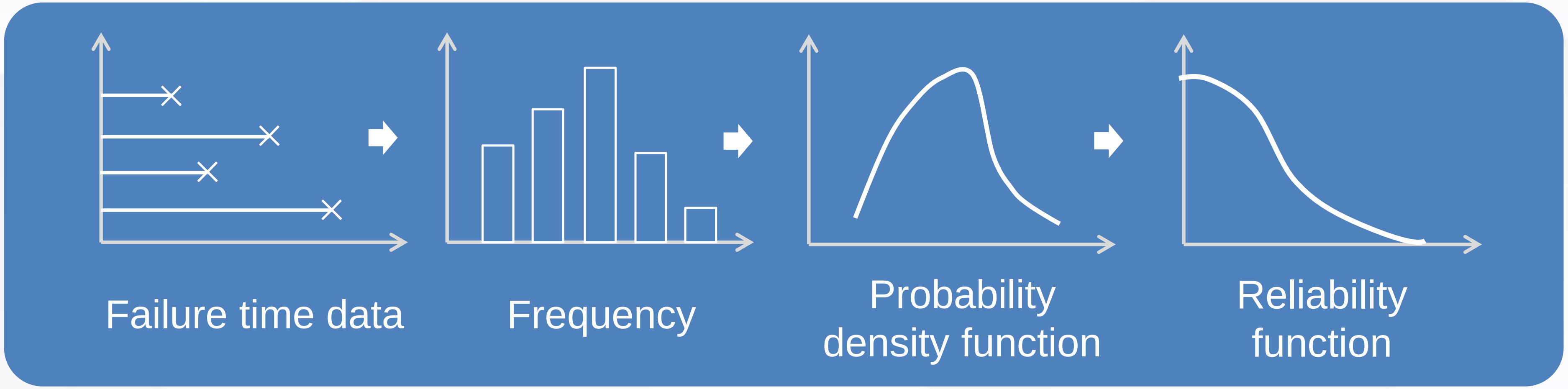
R. Lusser (1899-1969)

System reliability is the product of the reliability of each subsystem.

$$R_S = R_1 \cdot R_2 \cdot \ldots \cdot R_n$$

Classical probabilistic reliability metric

Black box method: Probabilistic metric based on failure data



- **Features：** The reliability is calculated using statistical methods
  This method doesn't separate aleatory and epistemic uncertainty
- **Shortage：** We must collect enough failure time data
  It is hard to indicate how to improve reliability

[1] W. Q. Meeker and L. Escobar, *Statistical methods for reliability data*. New York: Wiley, 1998.

Classical probabilistic reliability metric

White box method: Probabilistic metric based on physics of failure

- Physics-of-failure models (PoF models)

A PoF model is a mathematical model that quantifies the relationship between failure time or performance and product's features, such as material, structure, load, stress, etc. It is developed for one specific failure mechanism based on physics and chemistry theories.

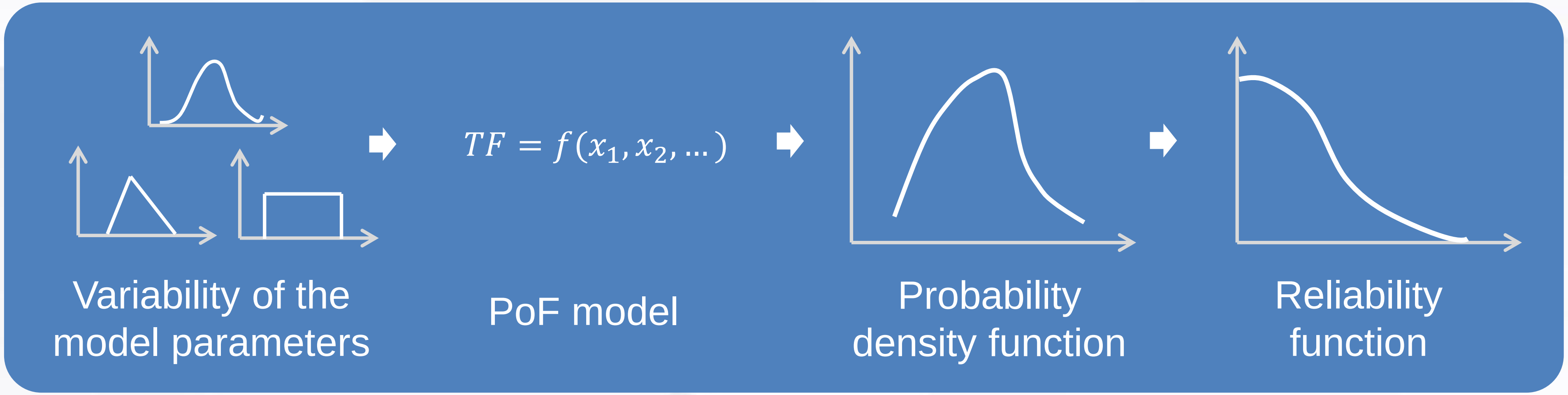
- A simple example – Archard's model (wear life model)

$$N = \frac{h_s H A}{\mu W_a L_m}$$

Failure time	N:	Wearing times
Material	H:	Hardness
	μ:	Dynamic friction coefficient
Structure	A:	Contact area of two wear surfaces
Load	W_a:	Contact pressure
Threshold	h_s:	The max acceptable wear volume

Classical probabilistic reliability metric

White box method: Probabilistic metric based on physics of failure



- **Features:** The failure is described by a deterministic model
  The uncertainty only comes from the variability of model parameters
  This method is able to measure reliability when there's few data
  The results can guide design improvements
- **Shortage:** The method may overestimate the reliability by ignoring epistemic uncertainty

[1] M. JW, *Reliability physics and engineering: Time-to-failure modeling*, 2nd ed. New York: Springer, 2013.

Classical probabilistic reliability metric

White box method: Source of epistemic uncertainty

Functional principle

Failure mechanism

PoF model

$$TF = f(x_1, x_2, \dots)$$

Variability of parameters

Lack of knowledge about the product function and failure mechanism → Model uncertainty

Lack of knowledge about the product working conditions → Parameter uncertainty

Reliability metric considering epistemic uncertainty

[1] T. Aven and E. Zio, Model output uncertainty in risk assessment, Int. J. Perform. Eng., 9(5):475-486, 2013.
[2] T. Bjerga, T. Aven and E. Zio, An illustration of the use of an approach for treating model uncertainties in risk assessment, Rel. Eng. Syst. Safety, 125:46-53, 2014.

Reliability metric considering EU

Imprecise probabilistic reliability metric

Bayes theory — Bayesian reliability
Evidence theory — Evidence reliability
Interval analysis — Interval reliability

Fuzzy reliability metric

Fuzzy theory — Fuzzy reliability

Reliability metric considering EU

Imprecise probabilistic reliability metric

Bayes theory — Bayesian reliability
Evidence theory — Evidence reliability
Interval analysis — Interval reliability

Posbist reliability metric

Possibility theory — Posbist reliability

Reliability metric considering EU

Imprecise probabilistic metric: Bayesian reliability

- Theoretical basis – Bayes theorem

Likelihood Function

Prior Distribution Function

（Subjective Information）

$$\boldsymbol{p}(\boldsymbol{\theta}|\boldsymbol{y}) = \frac{\boldsymbol{f}(\boldsymbol{y}|\boldsymbol{\theta})\boldsymbol{p}(\boldsymbol{\theta})}{\boldsymbol{m}(\boldsymbol{y})}$$

Posterior Distribution Function

Sampling density function

- How to consider EU?

Our knowledge on the failure process is reflected in the different forms of prior distribution.

---

[1] MS. Hamada, AG. Wilson, CS. Reese and HG. Martz, *Bayesian Reliability*, Spinger, 2008.

Reliability metric considering EU

Imprecise probabilistic metric: Bayesian reliability

- Method to obtain reliability

$$R(t) = \int_t^{\infty} f_T(\xi|\boldsymbol{\theta})d\xi$$

$f_T(t|\boldsymbol{\theta})$: pdf of failure time T

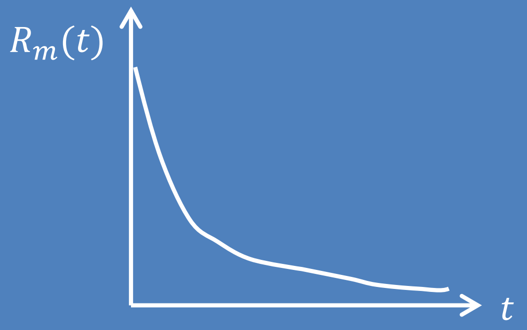
+

$p(\boldsymbol{\theta})$: prior distribution of parameter $\boldsymbol{\theta}$

+

$\boldsymbol{t}$: some failure time data

$p(\boldsymbol{\theta}|t)$: posterior distribution of $\boldsymbol{\theta}$

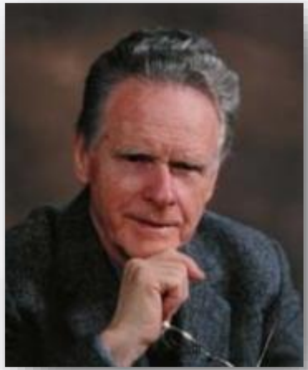


Use the median reliability $R_m(t)$ as the reliability index

[1] MS. Hamada, AG. Wilson, CS. Reese and HG. Martz, *Bayesian Reliability*, Spinger, 2008.

Reliability metric considering EU

Imprecise probabilistic metric: Evidence reliability



- Theoretical basis – Evidence theory
  - Proposed by A. Dempster and G. Shafer and refined by Shafer.
  - Use evidence to calculate Belief and Plausibility → Probability interval

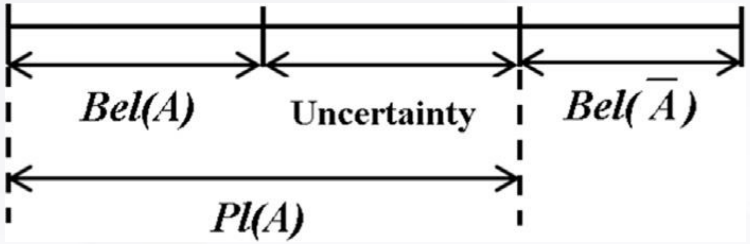


Fig. Belief and Plausibility

Bel: measures the evidence that supports A

Pl : measures the evidence that refutes A

$$Bel(A) \leq P(A) \leq Pl(A)$$

- How to consider EU?

Experts may set basic probability assignment (BPA) to different values of the model parameters based on experience or similar product information, reflecting the belief degree of the corresponding values.

[1] G. Shafer, *A mathematical theory of evidence*, Princeton: Princeton University Press, 1976.

Reliability metric considering EU

Imprecise probabilistic metric: Evidence reliability

- Method to obtain reliability

Construct a performance model $y = g(x_1, x_2, \ldots)$

↓

Identify the failure region $\{y < y_{th}\}$
Event A: system is working

↓

Define the frame of discernment and assign BPAs to possible values of parameters

↓

Calculate probability interval $[Bel(A), Pl(A)]$

For example, y represents output voltage and $y = g(x_1, x_2) = x_1^2 x_2/20$

Let $y_{th} = 1V$, then A ={y $\geq 1V$} denotes working state

$\Theta = \{[2,4] \times [2,4]\}$					
	Intervals	BPA		Intervals	BPA
x_1	[2.0, 2.5]	0.0478	x_2	[2.0, 2.5]	0.0478
	[2.5, 3.0]	0.4522		[2.5, 3.0]	0.4522
	[3.0, 3.5]	0.4522		[3.0, 3.5]	0.4522
	[3.5, 4.0]	0.0478		[3.5, 4.0]	0.0478

$0.5 \leq P(A) \leq 0.976$

[1] ZP. Mourelatos and J. Zhou, A design optimization method using evidence theory, *Journal of Mechanical Design*, 2006, 128(4): 901-908.

Reliability metric considering EU

Imprecise probabilistic metric: Interval reliability

■ Theoretical basis – Interval analysis

- Proposed by Ramon E. Moore.
- Calculate the interval of model output based on intervals of input parameters

Input parameter $x_L \leq x \leq x_U$ → Model $y = f(x)$ → Model output $y_L \leq y \leq y_U$

↑

Interval algorithm or optimization algorithm

■ How to consider EU?

The expert may give the upper and lower bounds of the model parameters based on experience or similar product information. Parameters can take any values within the given interval. The width of the interval reflects the degree of epistemic uncertainty.

[1] RE. Moore, *Methods and applications of interval analysis*. Philadelphia: Siam, 1979.

Reliability metric considering EU

Imprecise probabilistic metric: Interval reliability

- Method to obtain reliability

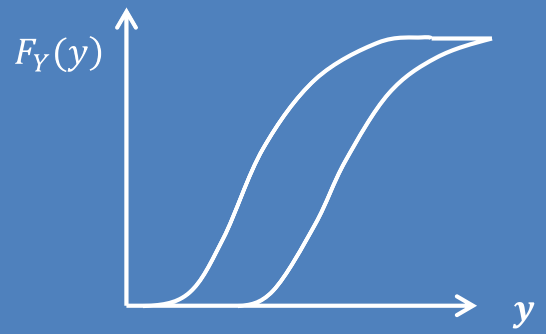
Construct a performance model
$y = f(x_1, x_2, \ldots)$

+

The upper and lower bounds of distributions are given by experts
$[\mu_{iL}, \mu_{iU}], [\sigma_{iL}, \sigma_{iU}], \ldots$

➡

Construct a p-box of y



Algorithm:
Cartesian product method[1]
Optimization method[2]

➡

$p = P(y \leq y_{th}) = F_Y(y_{th})$
Then we have $[\boldsymbol{p_L}, \boldsymbol{p_U}]$
and $[\boldsymbol{R_L}, \boldsymbol{R_U}]$

[1] DR. Karanki, HS. Kushwaha, AK. Verma et al. , Uncertainty analysis based on probability bounds (P-Box) approach in probabilistic safety assessment, *Risk Analysis*, 2009, 29(5): 662-675.
[2] H. Zhang, RL. Mullen, RL. Muhanna, Interval Monte Carlo methods for structural reliability, Structural Safety, 2010, 32(3): 183-190.

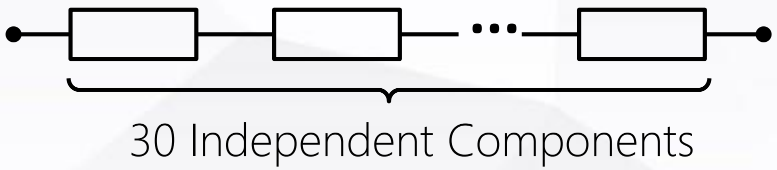
Reliability metric considering EU

Shortages of Imprecise probabilistic metric

- Interval extension problem

Example

Consider a series system composed of 30 components. Suppose that the reliability interval for each component is $[0.9,1]$. Then, the system's reliability metric will be $[0.9^{30}, 1^{30}] = [0.04,1]$,which is obviously too wide to provide any valuable information in practical applications.



- Disconnection between macro and micro

The metrics doesn't show the relationship between reliability and product design parameters. Therefore, their abilities to guide the improvement of products are very limited.

Reliability metric considering EU

Imprecise probabilistic reliability metric

Bayes theory — Bayesian reliability
Evidence theory — Evidence reliability
Interval analysis — Interval reliability
Fuzzy set theory — Fuzzy interval reliability

Posbist reliability metric

Possibility theory — Posbist reliability

Reliability metric considering EU

Posbist reliability metric

- Theoretical basis – Possibility theory

Possibility theory（Zadeh,1978）

In possibility theory, the possibility measure Π satisfies three axioms:

Axiom1. For the empty set $\emptyset$, $\Pi(\emptyset)=0$,

Axiom2. For the universal set Γ, $\Pi(\Gamma)=1$,

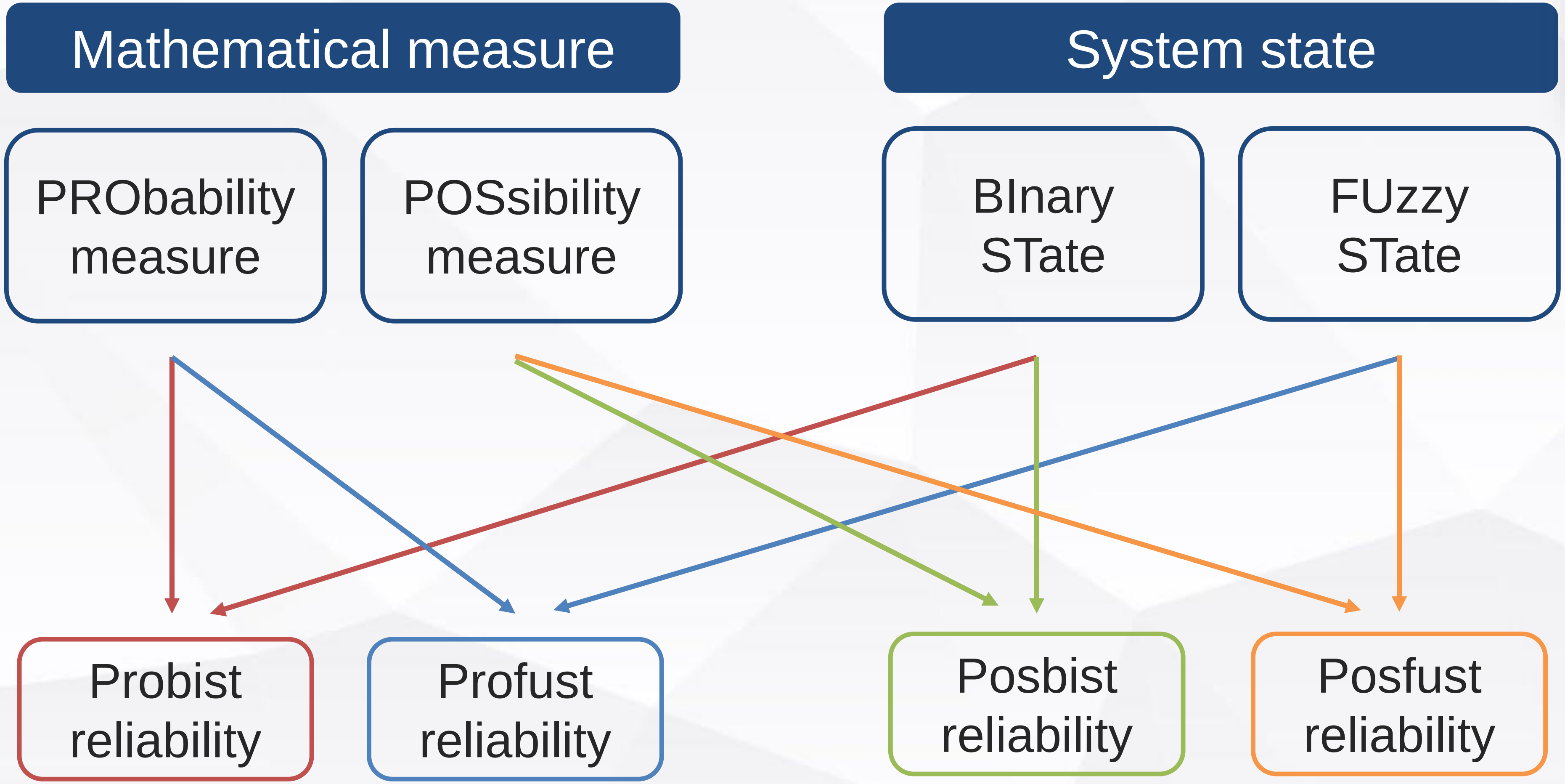
Axiom3. For any events Λ_1 and Λ_2 in the universal set Γ, there is

$$\Pi(\Lambda_1 \cup \Lambda_2)=\max(\Pi(\Lambda_1),\Pi(\Lambda_2))$$

[1] L.A. Zadeh, Fuzzy sets, *Information and Control*, 1965, 8: 338-353.

Reliability metric considering EU

Fuzzy reliability metric



[1] Kaiyuan Cai, *Introduction to fuzzy reliability*, Spinger, 1991.

Reliability metric considering EU

Posbist reliability metric

- **Basic assumption**
  - **Possibility assumption**

    System failure behavior can be characterized under possibility
  - **Binary-state assumption**

    The system demonstrates only two crisp states: functioning or failed
- **Definition**

Posbist Reliability (Cai , 1991)

Suppose the system failure time T is a fuzzy variable. Then the posbist reliability at time t is defined as the possibility measure that T is greater than t:

$$R(t) = \Pi(T \geq t)$$

- **How to consider EU?**

The failure time is modeled as a fuzzy variable, and the possibility distribution of failure time describes the epistemic uncertainty.

[1] Kaiyuan Cai, *Introduction to fuzzy reliability*, Spinger, 1991.

Reliability metric considering EU

Shortages of posbist reliability metric

- Non-duality

Example

Consider two exclusive events: Λ_1 ={The system is working}, Λ_2 ={The system fails}. Obviously, the universal set $\Gamma = \{\Lambda_1, \Lambda_2\}$. Then, we have the posbist reliability and posbist unreliability to be $R_{pos} = \Pi(\Lambda_1)$ and $\overline{R_{pos}} = \Pi(\Lambda_2)$.

According to Axiom 2 and Axiom 3, it can be proved that：

$$\boldsymbol{\Pi(\Gamma) = \Pi(\Lambda_1 \cup \Lambda_2) = \max\left(\Pi(\Lambda_1), \Pi(\Lambda_2)\right) = \max\left(R_{pos}, \overline{R_{pos}}\right) = 1}$$

Therefore, if $R_{pos} = 0.8$, then $\overline{R_{pos}} = 1$, and if $\overline{R_{pos}} = 0.8$, then $R_{pos} = 1$. This result is counterintuitive.

[1] Rui Kang, Qingyuan Zhang, Zhiguo Zeng, Enrico Zio, Xiaoyang Li. “Measuring reliability under epistemic uncertainty: Review on non-probabilistic reliability metrics ”. Chinese Journal of Aeronautics 29(3):571-579, 2016.

Outline

Research Background

Requirements Analysis

Theoretical Framework

Conclusion & Future

Requirements for reliability metric

Normality

A reliability metric must satisfy the normality principle, i.e., the sum of measurement of all states should be equal to 1. Specially, reliability plus unreliability must be 1.

This is mathematically consistent, also logically consistent. It can avoid the bug of fuzzy reliability.

[1] Rui Kang, Qingyuan Zhang, Zhiguo Zeng, Enrico Zio, Xiaoyang Li. "Measuring reliability under epistemic uncertainty: Review on non-probabilistic reliability metrics ". Chinese Journal of Aeronautics 29(3):571-579, 2016.

Requirements for reliability metric

Slow decrease

A reliability metric should be able to be used not only for the reliability evaluation of components and simple systems, but also for that of complex systems. When it is used for reliability calculation of the system, it cannot decrease as quickly as interval-based method, i.e., it should be able to compensate the conservatism in the component level.

[1] Rui Kang, Qingyuan Zhang, Zhiguo Zeng, Enrico Zio, Xiaoyang Li. “Measuring reliability under epistemic uncertainty: Review on non-probabilistic reliability metrics ”. Chinese Journal of Aeronautics 29(3):571-579, 2016.

Requirements for reliability metric

Multiscale analysis

A reliability metric must enable multiscale analysis. The bridge between reliability metric and product or system design elements can be established through multiscale analysis. This can provide more feedback on improving product or system reliability and avoids the embarrassment in statistical methods because statistical methods only give the results but don't know why.

[1] Rui Kang, Qingyuan Zhang, Zhiguo Zeng, Enrico Zio, Xiaoyang Li. “Measuring reliability under epistemic uncertainty: Review on non-probabilistic reliability metrics ”. Chinese Journal of Aeronautics 29(3):571-579, 2016.

Requirements for reliability metric

Uncertain information fusion

A reliability metric should be able to support the uncertain information fusion. The reliability information is available early in the design phase of a product. At this time, the degree of epistemic uncertainty is very high. As the design process advances, epistemic uncertainty will gradually decrease with a relative increase of aleatory uncertainty. The reliability metric must be able to integrate these different information.

---

[1] Rui Kang, Qingyuan Zhang, Zhiguo Zeng, Enrico Zio, Xiaoyang Li. “Measuring reliability under epistemic uncertainty: Review on non-probabilistic reliability metrics ”. Chinese Journal of Aeronautics 29(3):571-579, 2016.

Requirements for reliability metric

Theoretical Completeness

R1: Normality

R2: Slow decrease

Belief reliability theory

Engineering Practicability

R3: Multiscale analysis

R4: Information fusion

Outline

Research Background

Requirements Analysis

Theoretical Framework

Conclusion & Future

Preliminary about math theory

Belief reliability metric

Theoretical basis: Uncertainty theory

Uncertainty Theory（Liu,2007）

In uncertainty theory, the uncertainty measure $\mathcal{M}$ satisfies the following 4 axioms:

Axiom1. Normality axiom: For the universal set Γ, $\mathcal{M}\{\Gamma\} = 1$.

Axiom2. Duality axiom: For any event Λ, $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$.

Axiom3. Subadditivity axiom: For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots,$

$$\mathcal{M}\left\{\bigcup_{k=1}^{\infty} \Lambda_i\right\} \leq \sum_{k=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

Axiom4. Product axiom: For any uncertainty space $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k), k = 1,2, \ldots$,

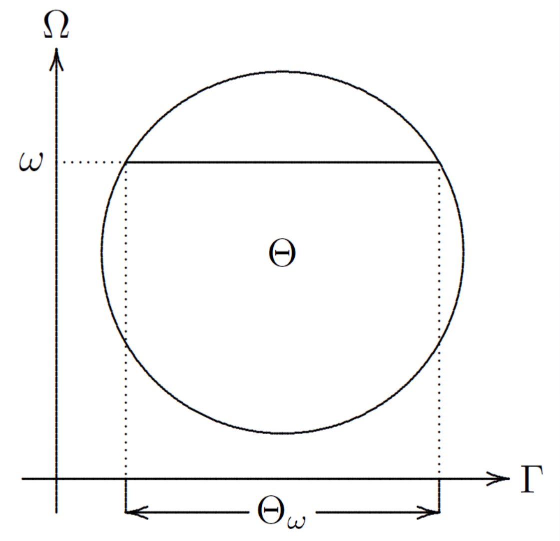
$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\}.$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k, k = 1,2, \ldots$

[1] Baoding Liu. *Uncertainty theory*., Spinger-Verlag, 2007.

General theoretical basis

Chance theory



Chance theory（Liu, 2013）

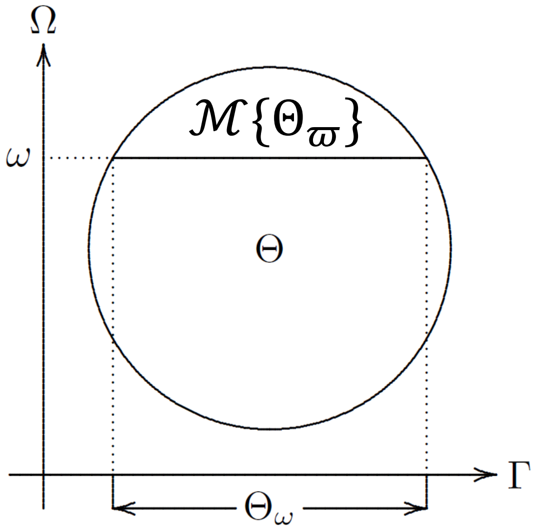
Chance theory defines chance measure Ch, which can be regarded as **a mixture of probability measure and uncertainty measure**.

Let $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{A}, \mathrm{Pr})$ be a chance space, and $\Theta \in \mathcal{L} \times \mathcal{A}$ is an event over this space. Then, the chance measure of Θ is defined to be：

$$\mathrm{Ch}\{\Theta\} = \int_0^1 \mathrm{Pr}\{\omega \in \Omega | \mathcal{M}\{\gamma \in \Gamma | (\gamma, \omega) \in \Omega\} \geq x\} dx$$

[1] Yuhan Liu. Uncertain random variables: a mixture of uncertainty and randomness. *Soft Computing*, 4(17): 625-634, 2013.

Chance theory



Chance measure（Liu, 2013)

Let $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{A}, \Pr)$ be a chance space, and $\Theta \in \mathcal{L} \times \mathcal{A}$ is an event over this space. Then, the chance measure of Θ is defined to be：

$$\mathrm{Ch}\{\Theta\} = \int_0^1 \Pr\{\omega \in \Omega | \mathcal{M}\{\gamma \in \Gamma | (\gamma, \omega) \in \Omega\} \geq x\} dx$$

Theorem

Let $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{A}, \Pr)$ be a chance space, then for any $\Lambda \in \mathcal{L}$ and $\mathrm{A} \in \mathcal{A}$：

$$\mathrm{Ch}\{\Lambda \times A\} = \mathcal{M}\{\Lambda\} \times \Pr\{A\}.$$

Especially we have $\mathrm{Ch}\{\emptyset\} = 0,\ \mathrm{Ch}\{\Gamma \times \Omega\} = 1.$

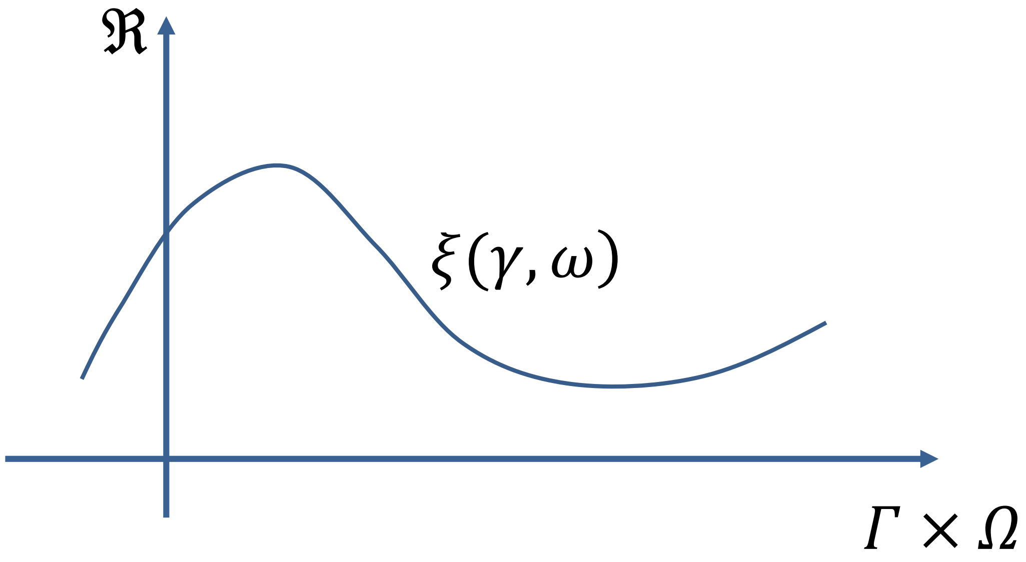
[1] Yuhan Liu. Uncertain random variables: a mixture of uncertainty and randomness. *Soft Computing*, 4(17): 625-634, 2013.

Chance theory

Basic concepts and theorems

Definition (Uncertain random variable)

An uncertain random variable is a function ξ from a chance space $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{A}, \Pr)$ to the set of real numbers such that $\{\xi \in B\}$ is an event in $\mathcal{L} \times \mathcal{A}$ for any Borel set B of real numbers.



- ξ can degenerate to a random variable if $\xi(\gamma, \omega)$ does not vary with γ.
- ξ can degenerate to an uncertain variable if $\xi(\gamma, \omega)$ does not vary with ω.

Chance theory

Basic concepts and theorems

Definition （Chance distribution）

Let ξ be an uncertain random variable, then its chance distribution is defined by

$$\Phi(x) = \mathrm{Ch}\{\xi \le x\}$$

for any $x \in \Re$. It can also degenerate to either probability or uncertainty distribution.

Definition （Expected value and variance）

Let ξ be an uncertain random variable, then its expected value is defined by

$$E[\xi] = \int_0^{+\infty} \mathrm{Ch}\{\xi \ge x\}dx - \int_{-\infty}^{0} \mathrm{Ch}\{\xi \le x\}dx,$$

provided that at least one of the two integrals is finite. Suppose ξ has an finite expected value e, the variance of ξ is defined as

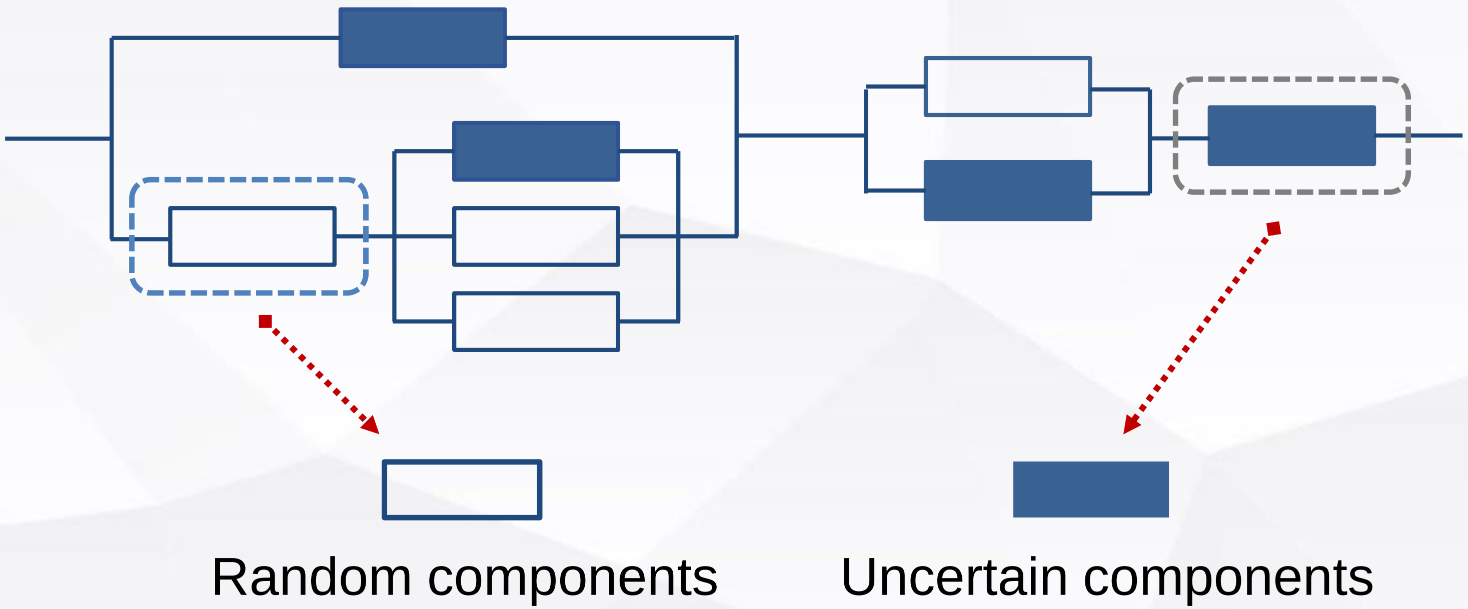
$$V[\xi] = E[(\xi - e)^2].$$

Concepts and definitions of belief reliability

Uncertain random systems

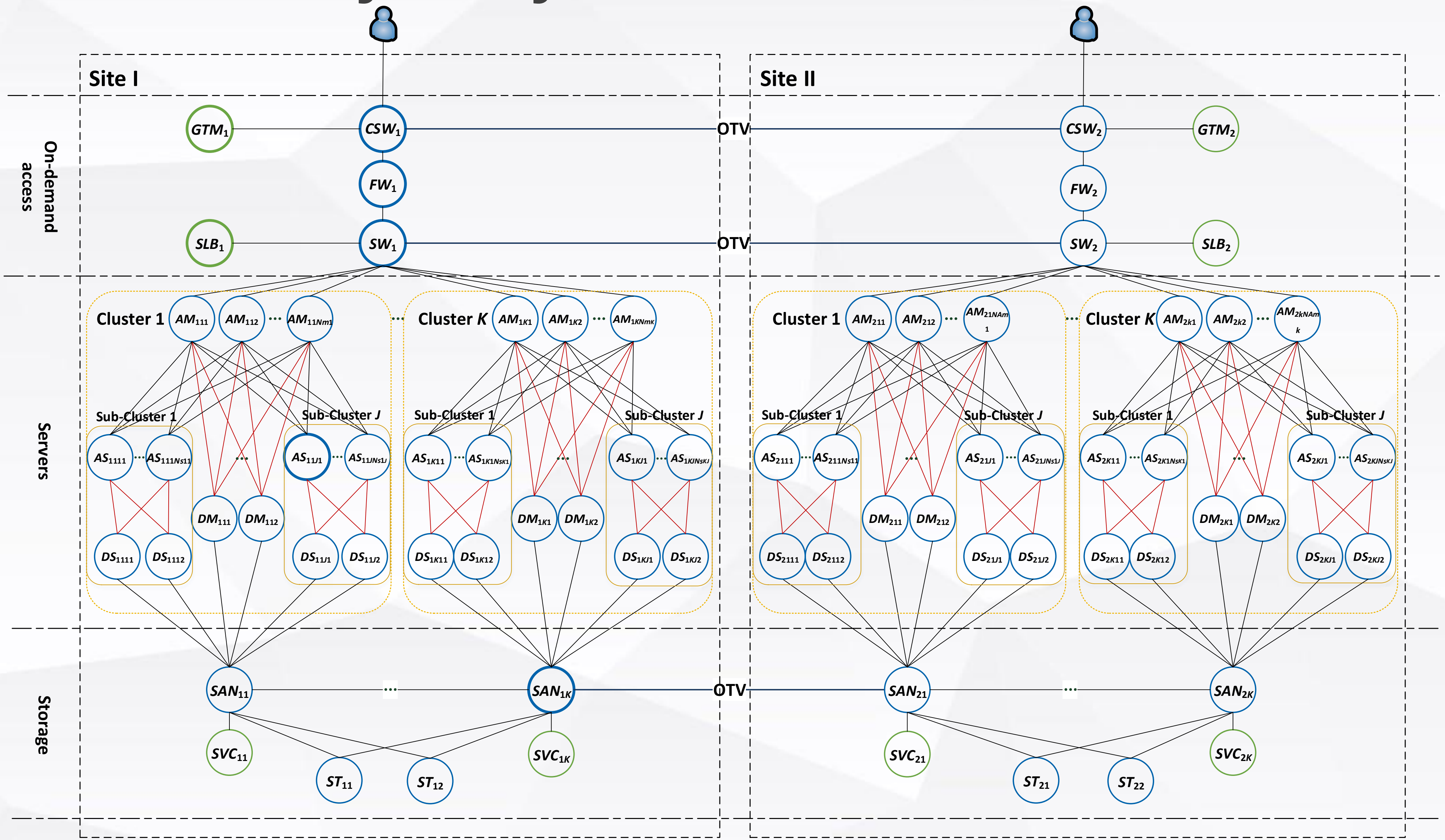
Definition: The system composed of uncertain and random components

- **Uncertain components:** Components affected by sever epistemic uncertainty. Their reliability can be described by uncertainty theory.
- **Random components:** Components mainly affected by aleatory uncertainty with sufficient failure data. Their reliability should be modeled by probability theory.



Real systems are usually uncertain random systems!

Belief reliability analysis of cloud data center



Belief reliability analysis of cloud data center

Parameter Setting - Certain Parameters

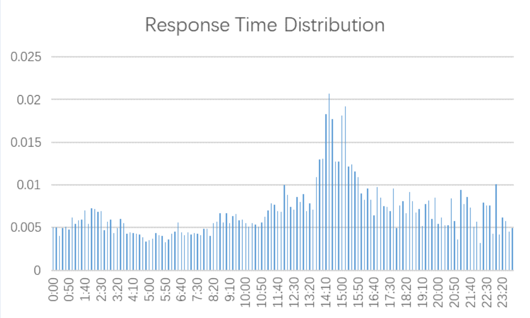
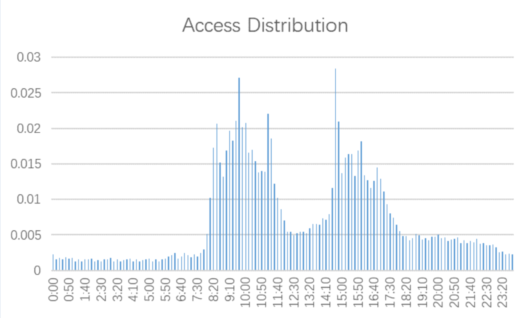
Parameters Related to the Design of CDC

Parameters	Setting
Function of Protocol and Routing Rules F_{PR}	According to the construction
Number of Clusters and Sub-Clusters K, J	
Number of Subtasks Y_{kA1}, Y_{kA1}, Y_{kD}	
Number of VMs for Each Physical Machine N_{VM}	
Number of Active Redundancy for Each Node N_R	
Number of Hot Standby for Each Node N_{HS}	

Belief reliability analysis of cloud data center

Parameter Setting - Uncertain Parameters

Parameters Related to the Operation and Maintenance of CDC

Parameters	Setting	Uncertainty
Working Probability *pr* Distribution Parameter of Processing Time λ_s	Evaluated through monitoring data 	Aleatory Uncertainty
Buffer Size *Q* Recovery Time Δt_r	Estimated by experts 	Epistemic Uncertainty
Distribution Parameter of Arrival Time λ_{ak}	Evaluated through monitoring data 	Aleatory Uncertainty

Definition and connotation of BR

Definition (Belief reliability)

Let a system state variable ξ be an uncertain random variable, and Ξ be the feasible domain of the system state. Then the belief reliability is defined as the chance that the system state is within the feasible domain, i.e.,

$$R_B = Ch\{\xi \in \Xi\}$$

Remark 1: ξ and Ξ

- The state variable ξ describe the system behavior (function or failure behavior), and the feasible domain Ξ is a reflection of failure criteria.
- ξ and Ξ can be relevant to time t, thus the belief reliability is a function of t, called belief reliability function $R_B(t)$.

Remark 2: Two special cases

- If the system is mainly affected by AU, ξ will degenerate to a random variable, and the belief reliability becomes $R_B^{(P)} = \Pr\{\xi \in \Xi\}$
- If the system is mainly affected by EU, ξ will degenerate to an uncertain variable, and the belief reliability becomes $R_B^{(U)} = \mathcal{M}\{\xi \in \Xi\}$

[1] Qingyuan Zhang, Rui Kang, Meilin Wen. Belief reliability for uncertain random systems. IEEE Transactions on Fuzzy Systems, 2018. (Online)

Definition and connotation of BR

Connotation 1: The state variable represents failure time

Example（Belief reliability based on failure time）

The system state variable can represent system failure time T which describes system failure behaviors. Therefore, the system belief reliability at t can be obtained by letting the feasible domain of T to be $\Xi = [t, +\infty)$, i.e.,

$$R_B(t) = \mathrm{Ch}\{T > t\}.$$

Two Special cases

If the system is mainly affected by AU, the failure time will be modeled as a random variable $T^{(P)}$, and we have $R_B(t) = R_B^{(P)}(t) = \Pr\{T^{(P)} > t\}$.

If the system is mainly affected by EU, the failure time will be modeled as an uncertain variable $T^{(U)}$, and we have $R_B(t) = R_B^{(U)}(t) = \mathcal{M}\{T^{(U)} > t\}$.

[1] Qingyuan Zhang, Rui Kang, Meilin Wen. Belief reliability for uncertain random systems. IEEE Transactions on Fuzzy Systems, 2018. (Online)

Definition and connotation of BR

Connotation 2: The state variable represents performance margin

Example（Belief reliability based on performance margin）

The system state variable can represent the performance margin m which describes system function behaviors. Let the feasible domain of m be $\Xi = (0, +\infty)$, and the system belief reliability can be written as:

$$R_B = \text{Ch}\{m > 0\}.$$

If we consider the degradation process of m, then the belief reliability function is

$$R_B(t) = \text{Ch}\{m(t) > 0\}.$$

$m(t)$ → $T = t_0 = \inf\{t \geq 0 | m(t) = 0\}$ → $R_B(t) = \text{Ch}\{m(t) > 0\}$

Uncertain random process

Failure time is just the first hitting time of uncertain random process

$$\begin{aligned} R_B(t) &= \text{Ch}\{m(t) > 0\} \\ &= \text{Ch}\{t_0 > t\} \\ &= \text{Ch}\{T > t\} \end{aligned}$$

[1] Qingyuan Zhang, Rui Kang, Meilin Wen. Belief reliability for uncertain random systems. IEEE Transactions on Fuzzy Systems, 2018. (Online)

Definition and connotation of BR

Connotation 3: The state variable represents function level

Example（Belief reliability based on function level）

The system state variable can represent the function level G which describes both system function and failure behaviors, then it can measure the reliability of multi-state systems. Assume the system has k different function levels with a lowest acceptable level of $G = s$. Let the feasible domain to be $\Xi = \{s, s+1, \cdots, k\}$, then the system belief reliability is

$$R_B = Ch\left\{G \in \{s, s+1, \cdots, k\}\right\}.$$

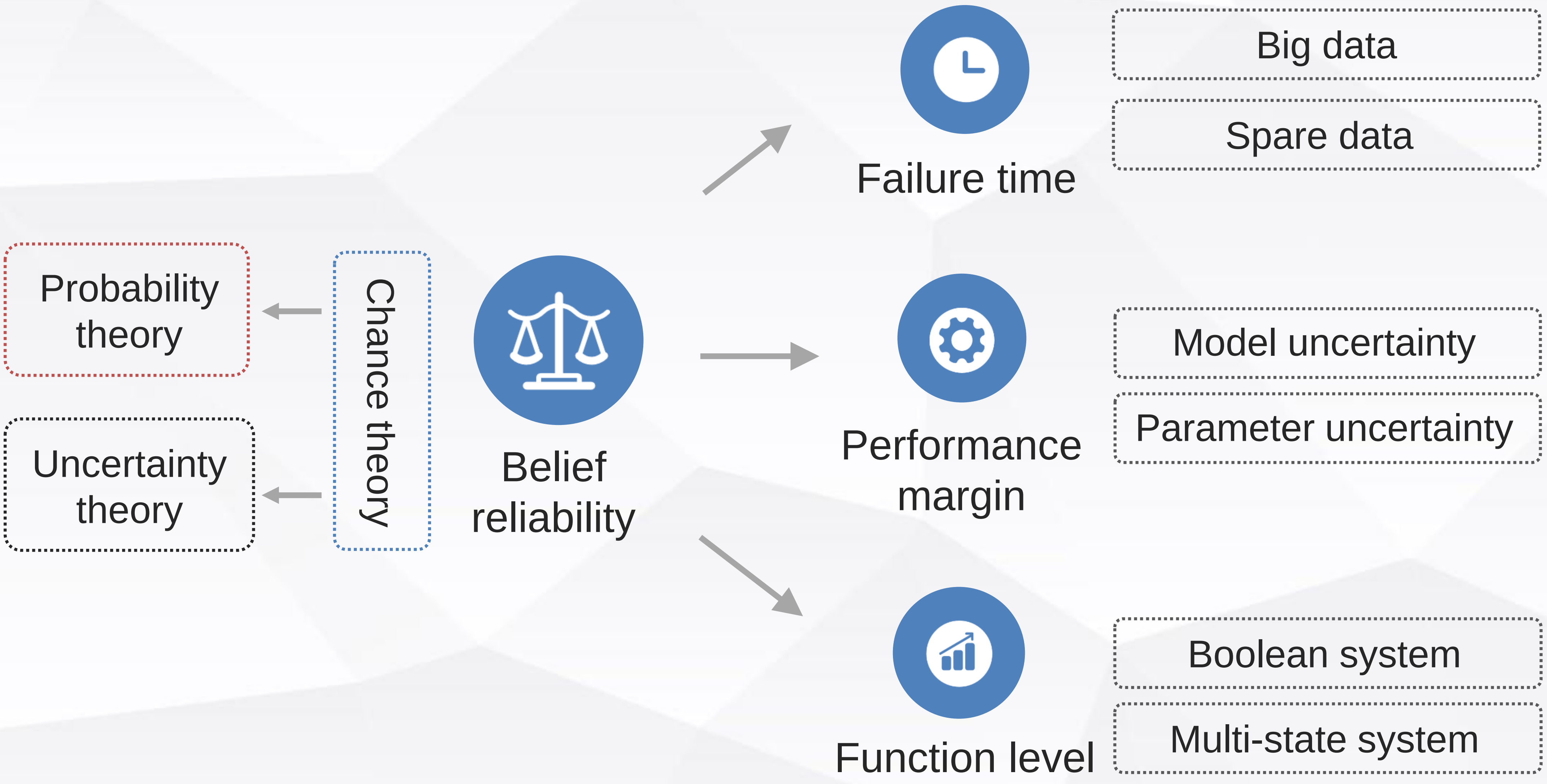
Special case

If the system has only two function levels, namely, complete failure with $G = 0$ and perfectly function with $G = 1$, then the belief reliability will be

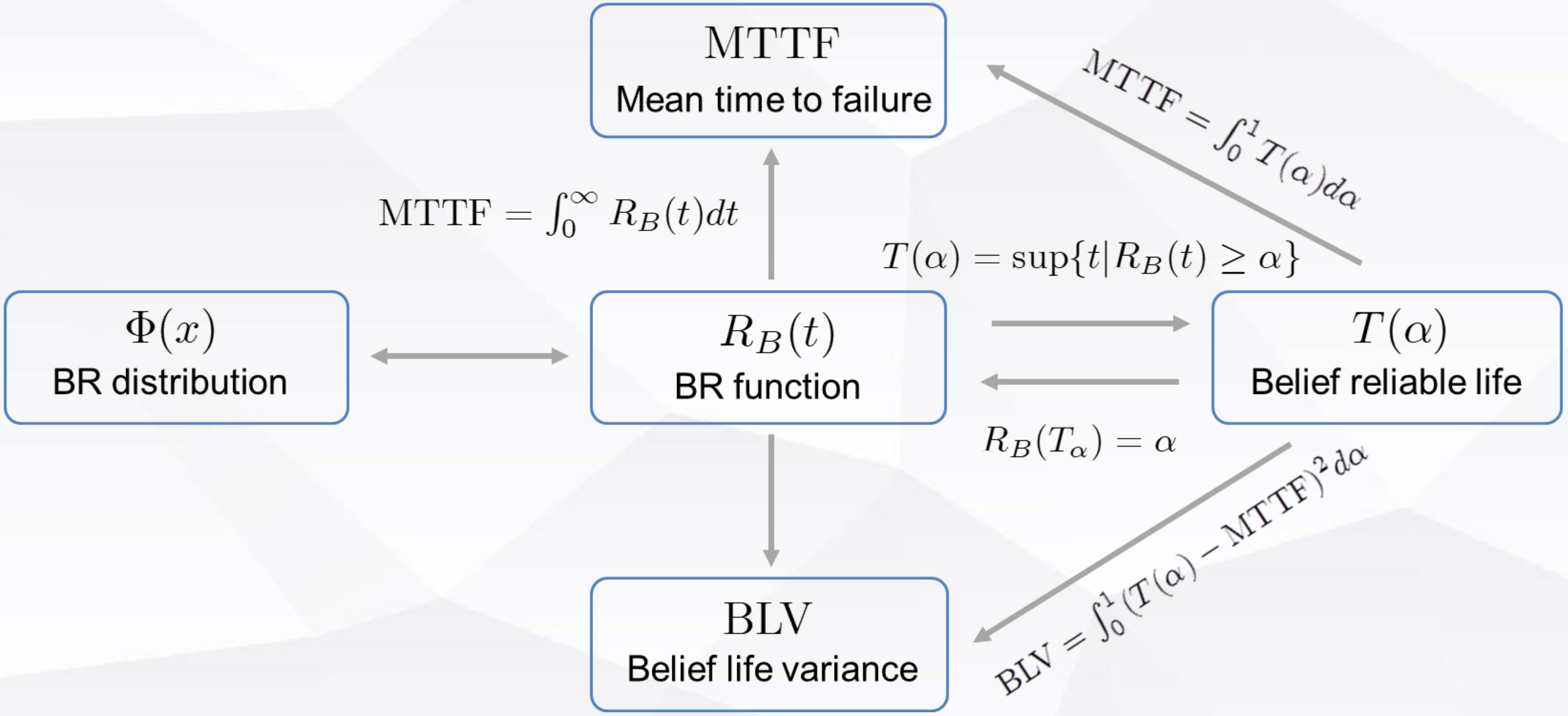
$$R_B = Ch\{G = 1\}.$$

[1] Qingyuan Zhang, Rui Kang, Meilin Wen. Belief reliability for uncertain random systems. IEEE Transactions on Fuzzy Systems, 2018. (Online)

Framework


Probability theory
Uncertainty theory
Chance theory
Belief reliability
Failure time
Big data
Spare data
Performance margin
Model uncertainty
Parameter uncertainty
Function level
Boolean system
Multi-state system

Belief reliability indexes



[1] Qingyuan Zhang, Rui Kang, Meilin Wen. Belief reliability for uncertain random systems. IEEE Transactions on Fuzzy Systems, 2018. (Online)

Some belief reliability indexes

Belief reliability distribution

Definition (Belief reliability distribution)

Assume that a system state variable ξ is an uncertain random variable, then the chance distribution of ξ, i.e.,

$$\Phi(x) = \mathrm{Ch}\{\xi \leq x\}$$

is defined as the belief reliability distribution.

If the state variable represents the system failure time, the BRD will be the chance distribution of T, denoted as $\Phi(t)$. It can degenerate to either probability or uncertainty distribution.

If the state variable represents the system performance margin, the RBD will be the chance distribution of m, denoted as $\Phi(x)$. It can degenerate to either probability or uncertainty distribution.

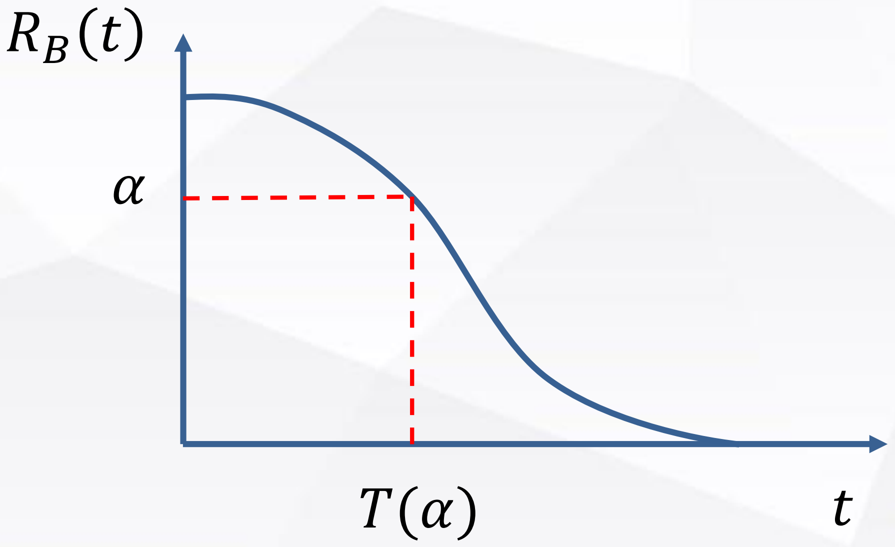
Some belief reliability indexes

Belief reliable life

Definition (Belief reliable life)

Assume the system failure time T is an uncertain random variable with a belief reliability function $R_B(t)$. Let α be a real number from $(0,1)$. The system belief reliable life $T(\alpha)$ is defined as

$$T(\alpha) = \sup\{t | R_B(t) \geq \alpha\}.$$



Some belief reliability indexes

Mean time to failure (MTTF)

Definition (Mean time to failure)

Assume the system failure time T is an uncertain random variable with a belief reliability function $R_B(t)$. The mean time to failure (MTTF) is defined as

$$\mathrm{MTTF} = E[T] = \int_0^{\infty} \mathrm{Ch}\{T > t\}dt = \int_0^{\infty} R_B(t)dt\,.$$

Theorem

Let $R_B(t)$ be a continuous and strictly decreasing function with respect to t at which $0 < R_B(t) < R_B(0) \leq 1$ and $\lim_{t\to+\infty} R_B(t) = 0$. Then we have

$$\mathrm{MTTF} = \int_0^{1} T(\alpha)d\alpha\,.$$

Some belief reliability indexes

Belief life variance (BLV)

Definition （Belief life variance）

Assume the system failure time T is an uncertain random variable and the mean time to failure is MTTF. The belief life variance (BLV) is defined as

$$\mathrm{BLV} = V[T] = E[(T - MTTF)^2] \; .$$

Theorem

Let the belief reliability function be $R_B(t)$, then the BLV can be calculated by

$$BLV = \int_0^{\infty} R_B(MTTF + \sqrt{t}) + 1 - R_B\left(MTTF - \sqrt{t}\right) dt.$$

Belief reliability for uncertain systems

Belief reliability for uncertain systems

Minimal cut set theorem for uncertain system

- Uncertain system is a system only composed of uncertain components. Its belief reliability can be calculated using minimal cut set theorem

Minimal cut set theorem

Consider a coherent uncertain system comprising n independent components with belief reliabilities $R_{B,i}^{(U)}(t), i = 1,2,\ldots,n$. If the system contains m minimal cut sets $C_1, C_2, \cdots, C_m$, then the system belief reliability is

$$R_{B,S}(t) = \bigwedge_{1\le i\le m} \bigvee_{j\in C_i} R_{B,j}^{(U)}$$

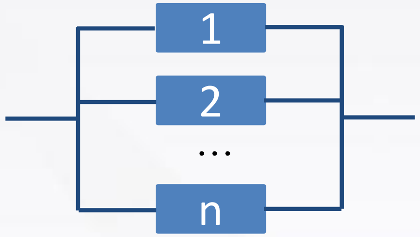
[1] Zhiguo Zeng, Rui Kang, Meilin Wen, Enrico Zio. Uncertainty theory as a basis for belief reliability. *Information Sciences*, 429: 26-36, 2018.

Belief reliability for uncertain systems

Some examples

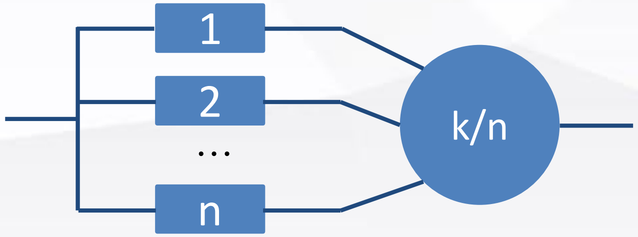
An uncertain series system has n minimal cut sets, i.e., $C_1 = \{1\}, C_2 = \{2\}, \cdots, C_n = \{n\}$. Then the belief reliability is

$$R_{B,S} = \min_{1 \le i \le n} \max_{j \in C_i} R_{B,j} = \min_{1 \le i \le n} R_{B,i}$$



An uncertain parallel system only has 1 minimal cut sets, i.e., $C_1 = \{1,2, \ldots n\}$. Then the belief reliability is

$$R_{B,S} = \max_{1 \le i \le n} R_{B,i}$$



An uncertain k-out-of-n system has C_n^{n-k+1} minimal cut sets and each set contains $n-k+1$ components arbitrary chosen from the n components. Assume $R_{B,1} \ge R_{B,2} \ge \cdots \ge R_{B,n}$, then belief reliability is

$$R_{B,S} = R_{B,k}$$

[1] Zhiguo Zeng, Rui Kang, Meilin Wen, Enrico Zio. Uncertainty theory as a basis for belief reliability. *Information Sciences*, 429: 26-36, 2018.

Belief reliability for uncertain systems

Uncertain fault tree analysis

- The belief reliability of uncertain system can be analyzed based on fault tree. The algorithm is an application of the minimal cut set theorem

Algorithm: BR analysis based on fault tree

1. Do a depth-first-search for the logic gates in the fault tree
2. For each logic gate, calculate the belief reliability for its output event:

$$R_{B,out} = \begin{cases} \bigwedge_{1\le i\le n} R_{B,in,i}\,, \text{for } an\ OR\ gate \\ \bigvee_{1\le i\le n} R_{B,in,i}\,, \text{for } an\ AND\ gate \end{cases}$$

3. $R_{B,S} \leftarrow R_{B,out,TE}$
4. Return $R_{B,S}$

[1] Zhiguo Zeng, Rui Kang, Meilin Wen, Enrico Zio. Uncertainty theory as a basis for belief reliability. *Information Sciences*, 429: 26-36, 2018.

Belief reliability for uncertain systems

An example: BR analysis of the left leading edge flap of F-18

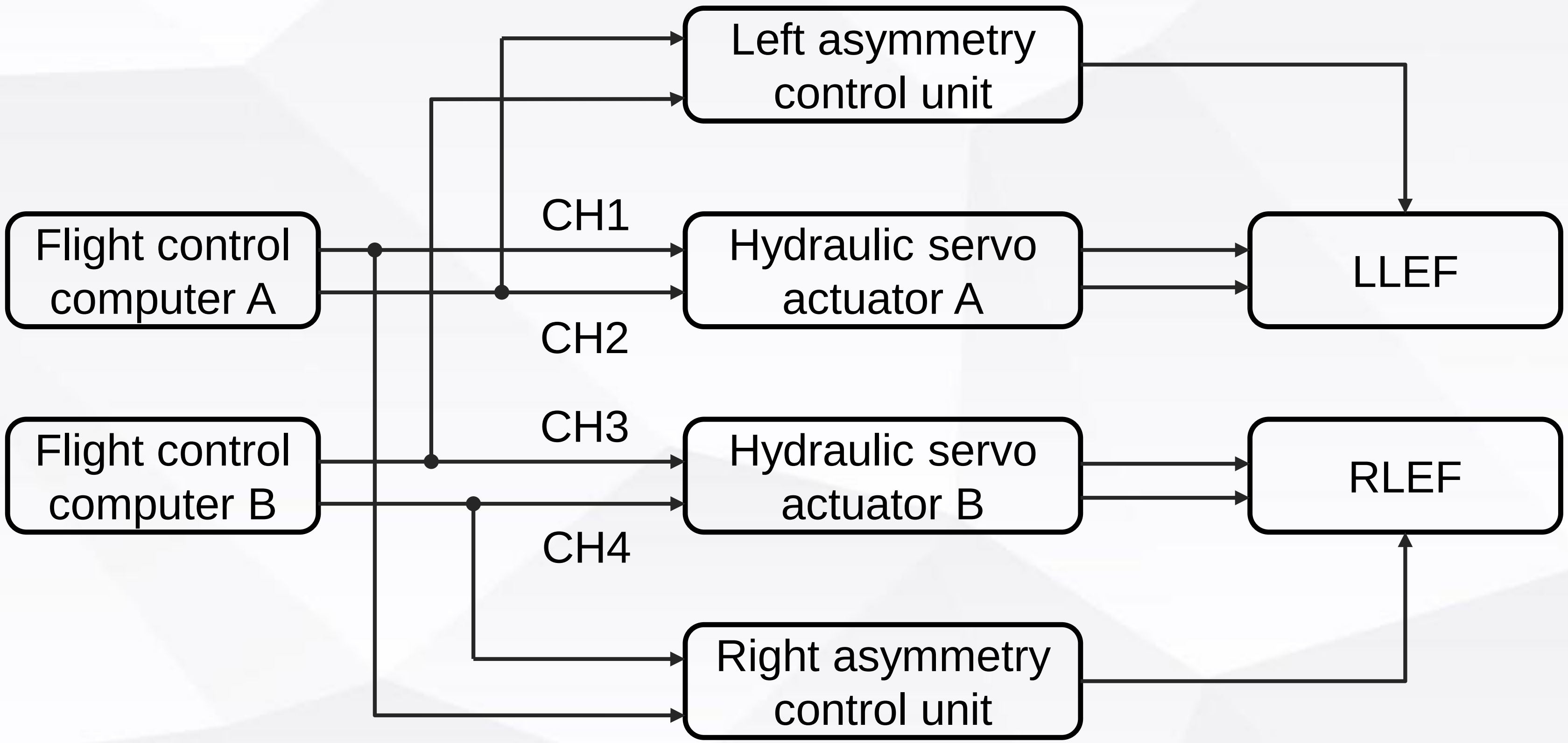


Fig. Schematic diagram of the F-18 left leading edge flap (LLEF)

[1] Zhiguo Zeng, Rui Kang, Meilin Wen, Enrico Zio. Uncertainty theory as a basis for belief reliability. *Information Sciences*, 429: 26-36, 2018.

Belief reliability for uncertain systems

An example: BR analysis of the left leading edge flap of F-18

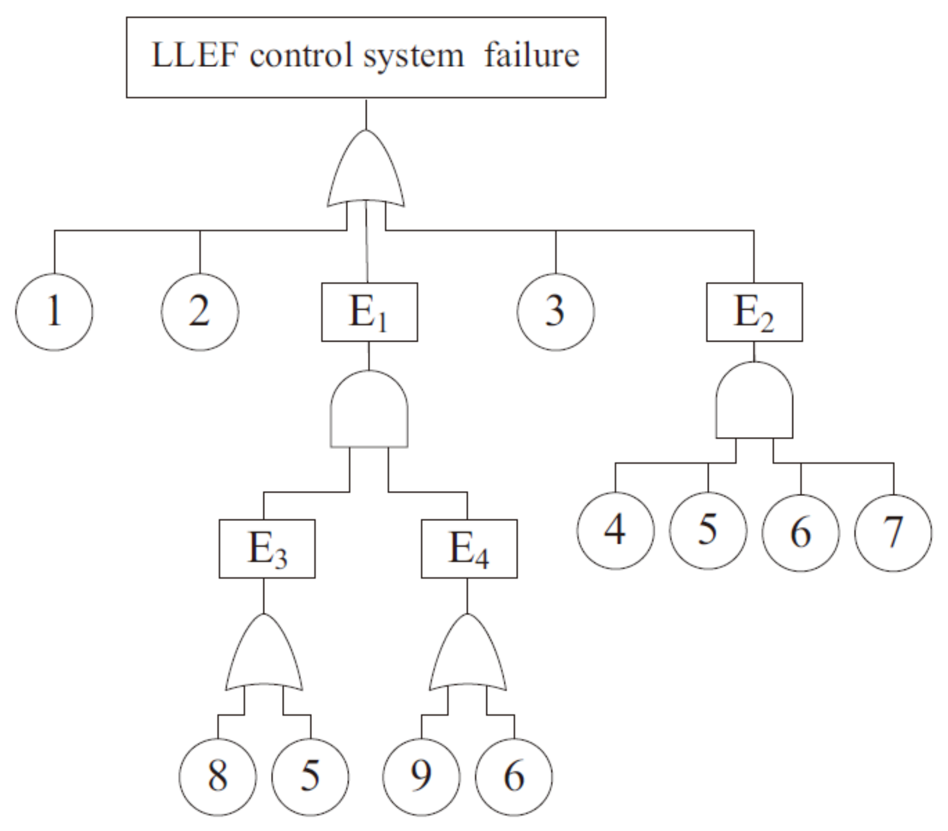


Fig. The fault tree of the F-18 LLEF

1 - HSA-A fail
2 - Left asymmetry control unit fail
3 - LLEF fail
4~7 - CH 1~4 fail
8 - FCC-A fail
9 - FCC-B fail

The system belief reliability is：

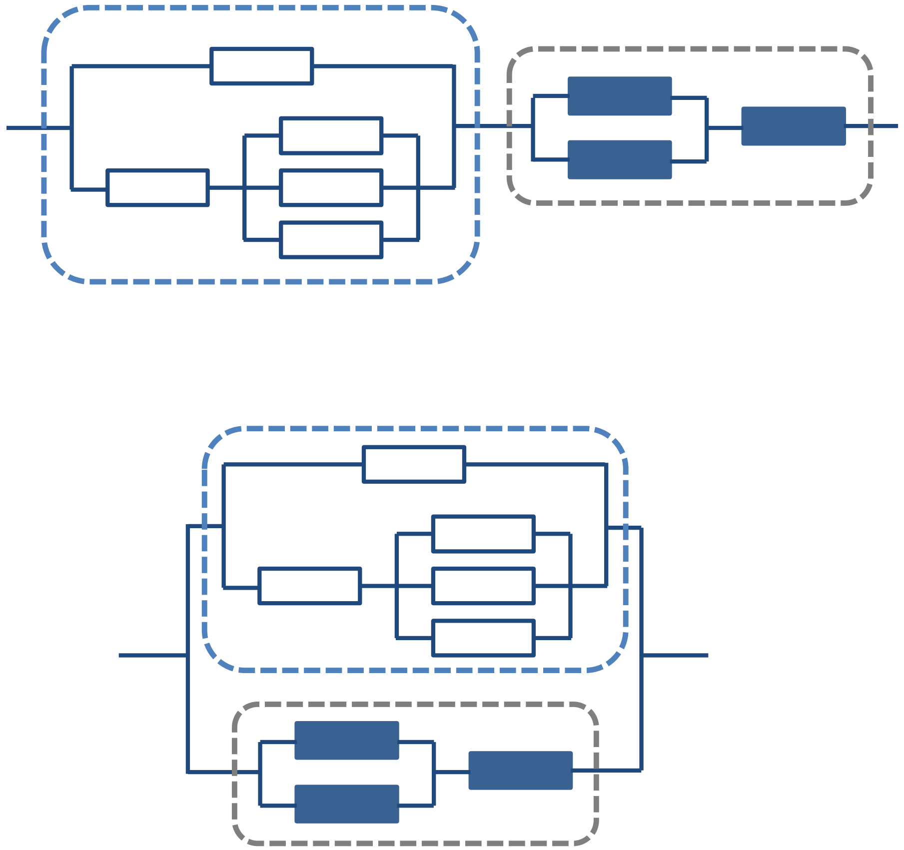
$$R_{B,S} = R_{B,1} \wedge R_{B,2} \wedge R_{B,3} \wedge \Big((R_{B,5} \wedge R_{B,8}) \vee (R_{B,6} \wedge R_{B,9}) \Big) \wedge (R_{B,4} \vee R_{B,5} \vee R_{B,6} \vee R_{B,7})$$

[1] Zhiguo Zeng, Rui Kang, Meilin Wen, Enrico Zio. Uncertainty theory as a basis for belief reliability. *Information Sciences*, 429：26-36, 2018.

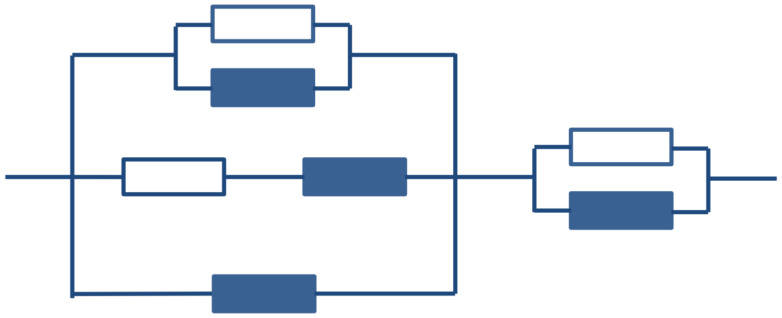
Belief reliability analysis for uncertain random systems

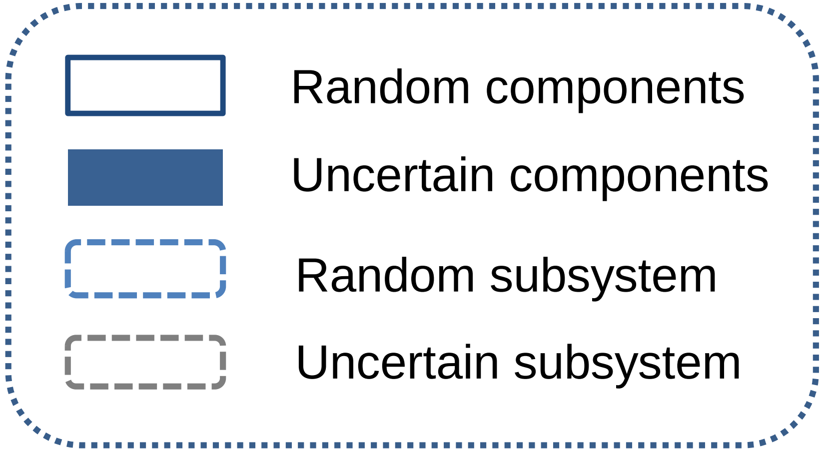
Simple and complex systems

Simple systems



Complex systems





Belief reliability formula for simple systems

Theorem (Simple system formula)

Assume an uncertain random system is simplified to be composed of a random subsystem with belief reliability $R_{B,R}^{(P)}(t)$ and an uncertain subsystem with belief reliability $R_{B,U}^{(U)}(t)$. If the two subsystems are connected in series, the system belief reliability will be

$$R_{B,S}(t) = R_{B,R}^{(P)}(t) \cdot R_{B,U}^{(U)}(t).$$

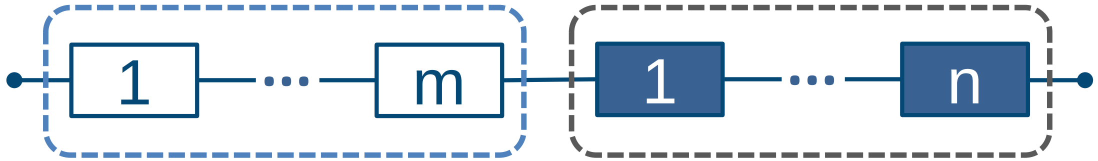
If the two subsystems are connected in parallel, the system belief reliability will be

$$R_{B,S}(t) = 1 - \left(1 - R_{B,R}^{(P)}(t)\right) \cdot \left(1 - R_{B,U}^{(U)}(t)\right).$$

Belief reliability formula for simple systems

Some examples

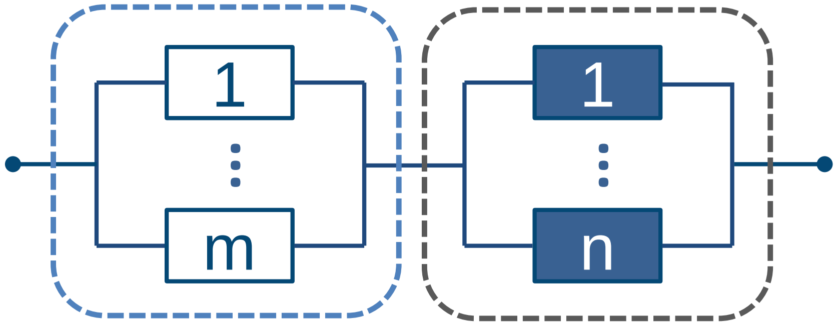
Series system



$R_{B,R}^{(P)}(t)$ $R_{B,U}^{(U)}(t)$

$$
\begin{aligned}
R_{B,S}(t) &= R_{B,R}^{(P)}(t) \cdot R_{B,U}^{(U)}(t) \\
&= \prod_{i=1}^{m} R_{B,i}^{(P)}(t) \cdot \bigwedge_{j=1}^{n} R_{B,j}^{(U)}(t).
\end{aligned}
$$

Parallel series system



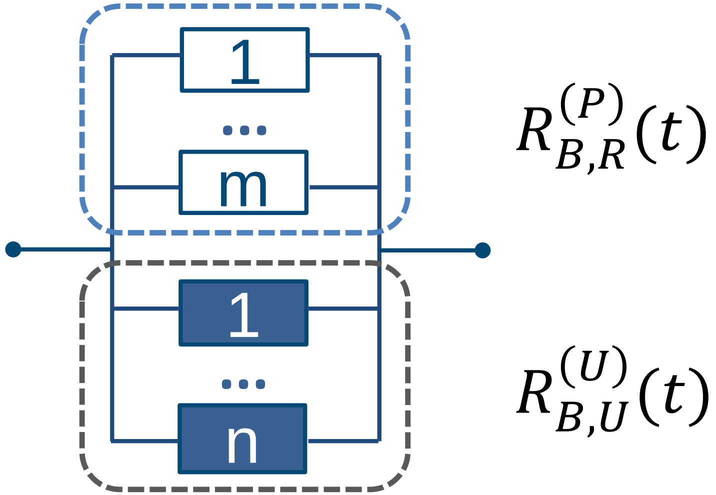
$R_{B,R}^{(P)}(t)$ $R_{B,U}^{(U)}(t)$

$$
\begin{aligned}
R_{B,S}(t) &= R_{B,R}^{(P)}(t) \cdot R_{B,U}^{(U)}(t) \\
&= \left(1 - \prod_{i=1}^{m}(1 - R_{B,i}^{(P)}(t))\right) \cdot \bigvee_{j=1}^{n} R_{B,j}^{(U)}(t).
\end{aligned}
$$

Belief reliability formula for simple systems

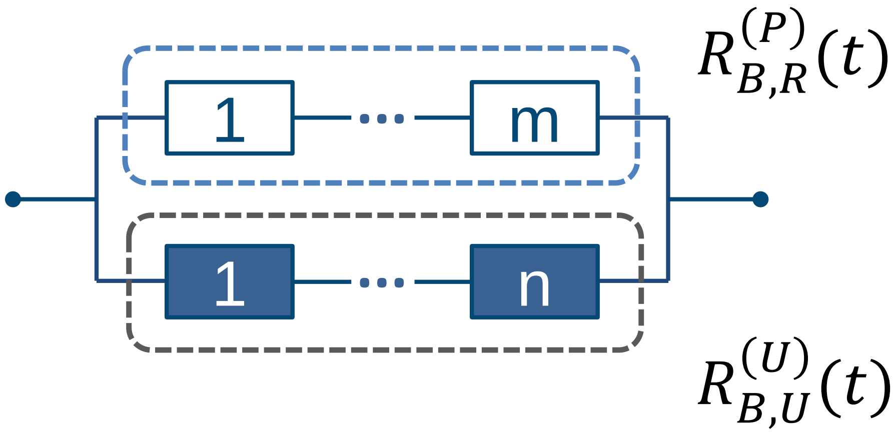
Some examples

Parallel system



$$R_{B,S}(t) = 1 - \left(1 - R_{B,R}^{(P)}(t)\right) \cdot \left(1 - R_{B,U}^{(U)}(t)\right)$$
$$= 1 - \left(\prod_{i=1}^{m}(1 - R_{B,i}^{(P)}(t))\right) \cdot \left(1 - \bigvee_{j=1}^{n} R_{B,j}^{(U)}(t)\right).$$

Series parallel system



$$R_{B,S}(t) = 1 - \left(1 - R_{B,R}^{(P)}(t)\right) \cdot \left(1 - R_{B,U}^{(U)}(t)\right)$$
$$= 1 - \left(1 - \prod_{i=1}^{m} R_{B,i}^{(P)}(t)\right) \cdot \left(1 - \bigwedge_{j=1}^{n} R_{B,j}^{(U)}(t)\right).$$

Belief reliability formula for complex systems

Theorem (Complex system formula, Wen & Kang, 2016)

Assume an uncertain random system is a Boolean system. The system has a structure function f and contains random components with belief reliabilities $R_{B,i}^{(P)}(t), i = 1,2,\cdots,m$ and uncertain components with belief reliabilities $R_{B,j}^{(U)}(t), j = 1,2,\cdots,n$. Then the belief reliability of the system is

$$R_{B,S}(t) = \sum_{(y_1,\cdots,y_m)\in\{0,1\}^m} \left(\prod_{i=1}^{m} \mu_i(y_i,t)\right) \cdot Z(y_1,y_2,\cdots,y_m,t),$$

where $Z(y_1,y_2,\cdots,y_m,t)$

$$= \begin{cases} \sup\limits_{f(y_1,\cdots,y_m,z_1,\cdots,z_n(t))=1} \min\limits_{1\leq j\leq n} \nu_j(z_j,t), & \text{if } \sup\limits_{f(y_1,\cdots,y_m,z_1,\cdots,z_n)=1} \min\limits_{1\leq j\leq n} \nu_j(z_j,t) < 0.5, \\ 1 - \sup\limits_{f(y_1,\cdots,y_m,z_1,\cdots,z_n)=0} \min\limits_{1\leq j\leq n} \nu_j(z_j,t), & \text{if } \sup\limits_{f(y_1,\cdots,y_m,z_1,\cdots,z_n)=1} \min\limits_{1\leq j\leq n} \nu_j(z_j,t) \geq 0.5, \end{cases}$$

$$\mu_j(y_i,t) = \begin{cases} R_{B,i}^{(P)}(t), & \text{if } y_i = 1, \\ 1 - R_{B,i}^{(P)}(t), & \text{if } y_i = 0, \end{cases} (i = 1,2,\cdots,m), \quad \nu_j(z_j,t) = \begin{cases} R_{B,i}^{(U)}(t), & \text{if } z_j = 1, \\ 1 - R_{B,i}^{(U)}(t), & \text{if } z_j = 0, \end{cases} (j = 1,2,\cdots,n).$$

A numerical case study

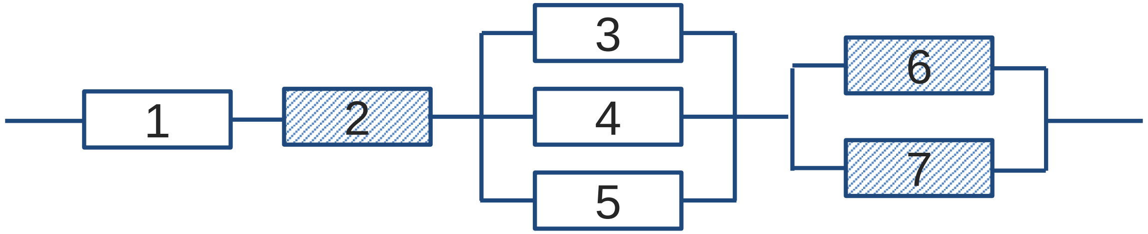


Table. Failure time distribution of components

No.	Components type	Failure time distribution
1,3,4,5	Random	$Exp(\lambda = 10^{-3}h^{-1})$
2	Uncertain	$L(500h, 3000h)$
6,7	Uncertain	$L(700h, 2700h)$

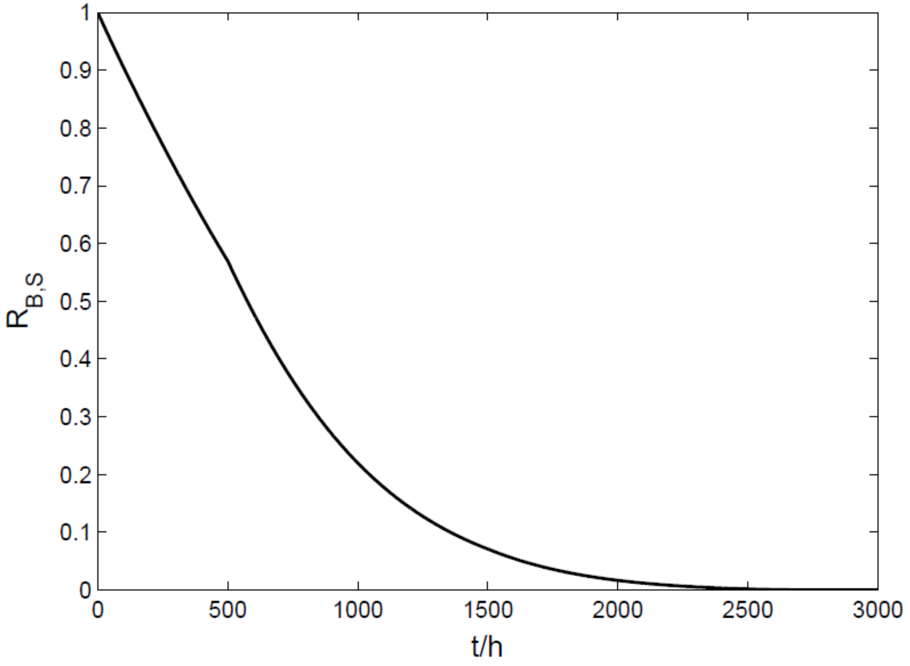


Figure. System belief reliability function

Belief reliability analysis for uncertain random components

Basic ideas

Performance margin model

$$m = g(x_1, x_2, \cdots, x_n)$$

Model uncertainty

The model may not precisely describe the function behavior, thus we need to add an uncertain random variable to quantify epistemic uncertainty.

$$m = g(x_1, x_2, \cdots, x_n, E)$$

Parameter uncertainty

Parameters in the model may be uncertain because of inherent variability and the uncertainty of real working conditions. Thus they are modeled as uncertain random variables.

$$m = g(x_1(\eta_1), x_2(\eta_2), \cdots, x_n(\eta_n))$$

BR analysis considering parameter uncertainty

Performance margin model

$m = g(x_1, x_2, \cdots, x_n)$

Model uncertainty

The model may not precisely describe the function behavior, thus we need to add an uncertain random variable to quantify epistemic uncertainty.

$m = g(x_1, x_2, \cdots, x_n, E)$

Parameter uncertainty

Parameters in the model may be uncertain because of inherent variability and the uncertainty of real working conditions. Thus they are modeled as uncertain random variables.

$m = g(x_1(\eta_1), x_2(\eta_2), \cdots, x_n(\eta_n))$

BR analysis with parameter uncertainty in margin model

Performance margin

Definition（Performance margin）

Assume the critical performance parameter of a system or a component is p , and its failure threshold is p_{th}, i.e., the system or the component will fail when $p > p_{th}$. Then the performance margin is defined as：

$$m = p_{th} - p$$

Remark：

1. The system or the component will be working when $m > 0$, and fail when $m < 0$.
2. Considering the parameter uncertainty of performance parameter and its threshold, there will be several cases:
   - p and p_{th} are both random variables
   - p and p_{th} are both uncertain variables
   - p is a random variable and p_{th} is an uncertain variable
   - p_{th} is a random variable and p is an uncertain variable

---

[1] Qingyuan Zhang, Rui Kang, Meilin Wen, Tianpei Zu. A performance-margin-based belief reliability model considering parameter uncertainty. ESREL 2018.

BR analysis with parameter uncertainty in margin model

Case 1: p and p_{th} are both uncertain variables

Theorem 1

Suppose the system critical performance parameter p and its associated failure threshold p_{th} are both uncertain variables, and their uncertainty distributions are $\Phi(x)$ and $\Psi(x)$, respectively. Then the system belief reliability will be:

$$R_B = \sup_{y \in \Re} \left(\Phi(y) \wedge (1 - \Psi(y)) \right).$$

A special case

If p_{th} is a constant, then the belief reliability will be: $R_B = \Phi(p_{th})$.

[1] Qingyuan Zhang, Rui Kang, Meilin Wen, Tianpei Zu. A performance-margin-based belief reliability model considering parameter uncertainty. ESREL 2018.

BR analysis with parameter uncertainty in margin model

Case 2: p is random and p_{th} is uncertain

Theorem 2

Suppose the system critical performance parameter p is a random variable with a probability distribution $\Phi(x)$, and the failure threshold p_{th} is an uncertain variable with an uncertainty distribution $\Psi(x)$. Then the system belief reliability is：

$$R_B = \int_{-\infty}^{+\infty} 1 - \Psi(y)\, d\Phi(y)$$

Case 3: p is uncertain and p_{th} is random

Theorem 3

Suppose the system critical performance parameter p is an uncertain variable with an uncertainty distribution $\Phi(x)$, and the failure threshold p_{th} is a random variable with a probability distribution $\Psi(x)$. Then the system belief reliability is：

$$R_B = \int_{-\infty}^{+\infty} \Phi(y)\, d\Psi(y)$$

BR analysis with parameter uncertainty in margin model

Case study: Belief reliability analysis of a contact recording head

$$V = k_s \cdot L_s \cdot W \cdot \left(\frac{L}{L_s}\right)^{1-a} \left(\frac{B}{b}\right)^{a}$$

Input parameters	Value or distribution	Input parameters	Value or distribution
Specific wear amounts k_s	$2.55 \times 10^{-20}\ (m^2/N)$	Sliding width B	$0.015(m)$
Running-in coefficient a	0.39	Contact area A	$10^{-8}(m^2)$
Standard sliding distance L_s	$1000(m)$	Head width b	$10^{-4}(m)$
Total sliding distance L	$3.6 \times 10^{6}(m)$		
Contact load W	$W \sim \mathcal{N}(\mu = 0.7, \sigma = 0.03)(mN)$		

The uncertainty distribution of V: $V \sim \mathcal{N}(\mu_V = 1.8606, \sigma_V = 0.07974)(10^{-17}m^3)$
The uncertainty distribution of V_{th} is estimated to be: $V_{th} \sim \mathcal{L}(a = 2, b = 2.5)(10^{-17}m^3)$

$$R_B = \sup_{x \in \Re} \left(\Phi_V(x) \wedge (1 - \Phi_{V_{th}}(x)) \right) = 0.97078$$

BR analysis with both model and parameter uncertainties

Performance margin model

$$m = g(x_1, x_2, \cdots, x_n)$$

Model uncertainty

The model may not precisely describe the function behavior, thus we need to add an uncertain random variable to quantify epistemic uncertainty.

$$m = g(x_1, x_2, \cdots, x_n, E)$$

Parameter uncertainty

Parameters in the model may be uncertain because of inherent variability and the uncertainty of real working conditions. Thus they are modeled as uncertain random variables.

$$m = g(x_1(\eta_1), x_2(\eta_2), \cdots, x_n(\eta_n))$$

Outline

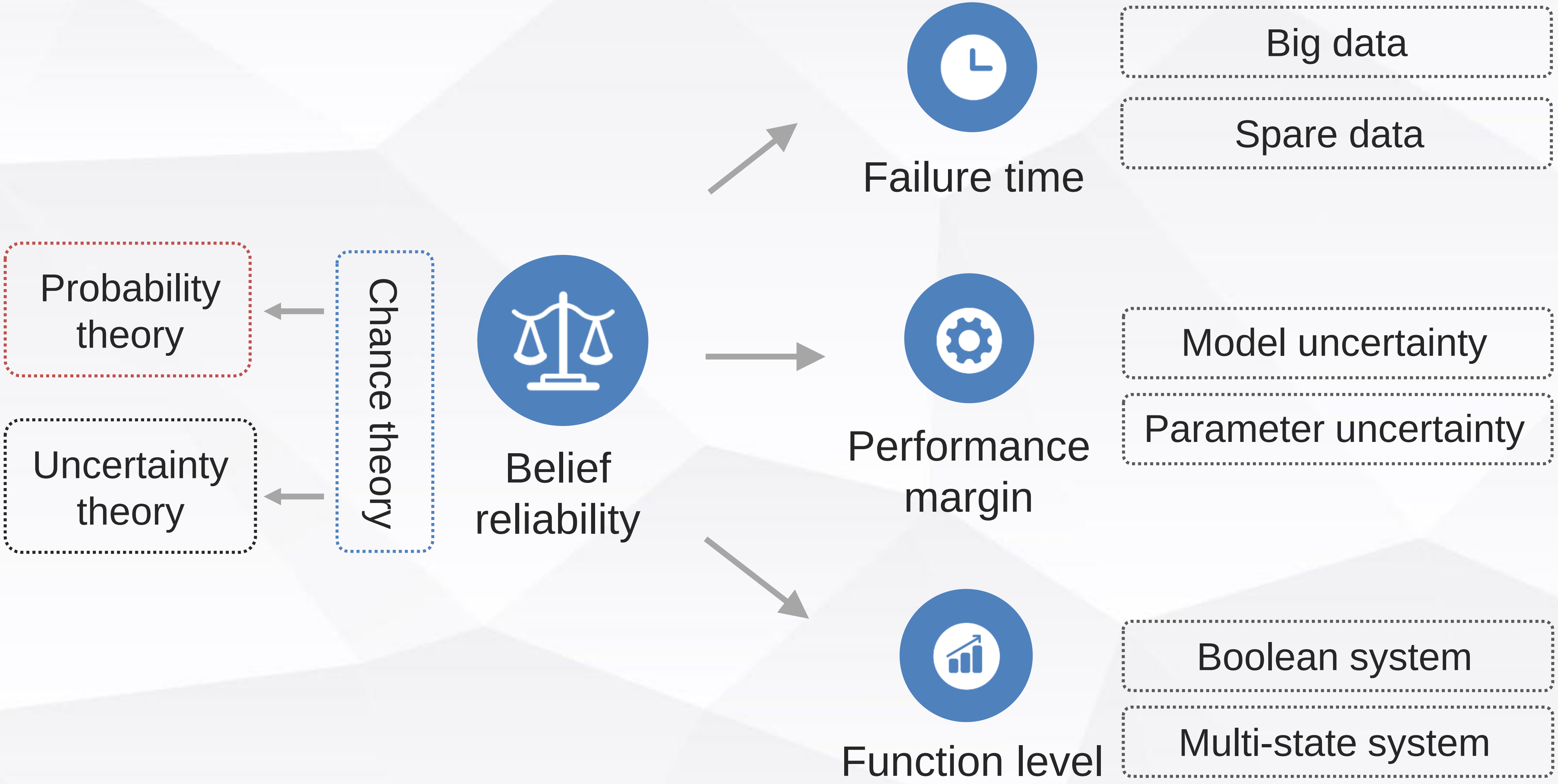
Research Background

Requirements Analysis

Theoretical Framework

Conclusion & Future

Conclusion


Probability theory
Uncertainty theory
Chance theory
Belief reliability
Failure time
Big data
Spare data
Performance margin
Model uncertainty
Parameter uncertainty
Function level
Boolean system
Multi-state system

References

Journal papers

[1] Qingyuan Zhang, Rui Kang, Meilin Wen. Belief reliability for uncertain random systems. *IEEE Transactions on Fuzzy Systems*, 2018. **(Q1, IF:8.415)** (Online)

[2] Zhiguo Zeng, Rui Kang, Meilin Wen, Enrico Zio. Uncertainty theory as a basis for belief reliability. *Information Sciences*, 429: 26-36, 2018. **(Q1, IF:4.305)**

[3] Meilin Wen, Tianpei Zu, Miaomiao Guo, Rui Kang, Yi Yang. Optimization of spare parts varieties based on stochastic DEA model. *IEEE Access*, 2018. **(Q1, IF:3.557)** (Online)

[4] Tianpei Zu, Rui Kang, Meilin Wen, Qingyuan Zhang. Belief Reliability Distribution Based on Maximum Entropy Principle . *IEEE Access*, 6(1): 1577-1582, 2017. **(Q1, IF:3.557)**

[5] Zhiguo Zeng, Rui Kang, Meilin Wen, Enrico Zio. A model-based reliability metric considering aleatory and epistemic uncertainty. *IEEE Access*, 5: 15505-15515, 2017. **(Q1, IF:3.557)**

[6] Qingyuan Zhang, Zhiguo Zeng, Enrico Zio, Rui Kang. Probability box as a tool to model and control the epistemic uncertainty in multiple dependent competing failure processes. *Applied Soft Computing*, 2017, 56: 570-579. **(Q1, IF:3.907)**

[7] Meilin Wen, Qiao Han, Yi Yang, Rui Kang, Uncertain Optimization Model for Multi-echelon Spare Parts Supply System, *Applied Soft Computing*, 2017, 56:646-654. **(Q1, IF:3.907)**

References

Journal papers

[8] Tianpei Zu, Meilin Wen, Rui Kang. An optimal evaluating method for uncertainty metrics in reliability based on uncertain data envelopment analysis. *Microelectronics Reliability*, 2017, 75: 283-287.

[9] Qingyuan Zhang, Rui Kang, Meilin Wen. A new method of level-2 uncertainty analysis in risk assessment based on uncertainty theory. *Soft Computing*, 2017. (Online)

[10] Rui Kang, Qingyuan Zhang, Zhiguo Zeng, Enrico Zio, Xiaoyang Li. Measuring reliability under epistemic uncertainty: A review on non-probabilistic reliability metrics. *Chinese Journal of Aeronautics*. 2016, 29(3): 571-579.

[11] Meilin Wen, Rui Kang, Reliability analysis in uncertain random system. *Fuzzy Optimization and Decision Making*. 2016, 15(4): 491-506.

[12] Zhiguo Zeng, Meilin Wen, Rui Kang. Belief reliability: a new metrics for products' reliability. *Fuzzy Optimization and Decision Making*. 2013, 12(1): 15-27.

[13] Xiaoyang Li, Jipeng Wu, Le Liu, Meilin Wen, Rui Kang. Modeling accelerated degradation data based on the uncertain process. *IEEE Transactions on Fuzzy Systems*, 2018. (Under Review)

[14] Tianpei Zu, Rui Kang, Meilin Wen. Modeling epistemic uncertainty by technology readiness levels. *Reliability Engineering and System Safety*, 2018. (Under Review)

Future

Abstract Objects

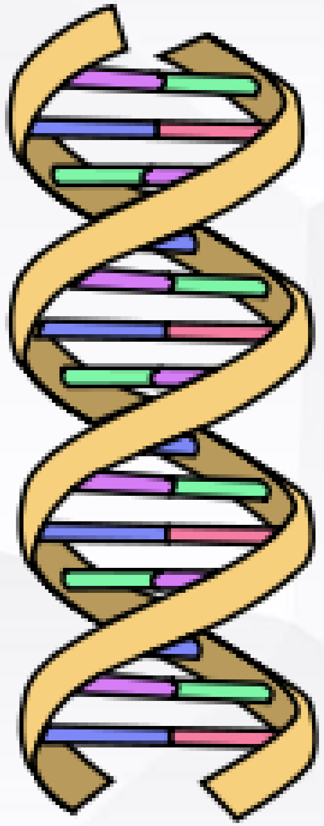
Cyber Physics Social System

Cyber Physics System

Network

Software

Hardware



Methodology

Failure/Fault Prevention

Failure/Fault Diagnosis

Failure/Fault Prognosis

Failure/Fault Control

Failurology

Recognize Failure Rules & Identify Failure Behaviors

关键基础设施可靠性安全性研究中心
Center for Resilience and Safety of Critical Infrastructures

Thank you!

kangrui@buaa.edu.cn

北京航空航天大学
BEIHANG UNIVERSITY